1. a) Write down the expression for the branch of \( \log z \) in the slit plane \( U = C \setminus \{ y i : y < 0 \} \), s.t. \( \log 1 = 0 \). Calculate \( \log(-1-i) \) for this branch.

b) Calculate \( \int_C \frac{dz}{z} \), where \( C \) is a curve in \( U \) beginning at 1 and ending at \(-1-i\).

2. Expand \( \frac{1}{(z+1)(z-3)} \) into a power series centered at 0. Where does it converge?

3. Show that the function
\[
\delta(z) = \begin{cases} 
\frac{e^{-z} - 1 + z}{z^2}, & z \neq 0 \\
\frac{1}{2}, & z = 0
\end{cases}
\]
is entire.

4. Calculate \( \int_{C} \frac{e^{is}}{s+\pi} \, ds \), where \( |s| = 2\pi \).
1. a) \( \log z = \log |z| + i \arg z \), where \(-\frac{\pi}{2} < \arg z < \frac{3\pi}{2} \) (so \( \arg 1 = 0 \))

\[
\log(-1-i) = \log \sqrt{2} + i \frac{s\pi}{4}.
\]

b) \( \int \frac{dz}{z} = \log z \bigg|_{-1}^{1-i} = \log(-1-i) - \log 1 = \log \sqrt{2} + i \frac{s\pi}{4} \)

2. \( \frac{1}{(z+1)(z-3)} = -\frac{1}{4} \left( \frac{1}{1+z} + \frac{1}{3-z} \right) = \)

\[=-\frac{1}{4} \sum_{n=0}^{\infty} (-1)^n z^n - \frac{1}{4} \sum_{n=0}^{\infty} \frac{z^n}{3^{n+1}}, \]

which converges for \( |z| < 1 \).

[One can also write Laurent expansions in the annulus \( \{2 < |z| < 3\} \) and in \( \{ |z| > 3\} \).]
3. \[ e^{-z} = 1 - z + \frac{z^2}{2} - \frac{z^3}{3!} + \ldots = \sum_{n=0}^{\infty} \frac{(1)^n}{n!} \frac{z^n}{n!} \]

\[ e^{-z} - 1 + z = \frac{z^2}{2} - \frac{z^3}{3!} + \frac{z^4}{4!} + \ldots = \sum_{n=2}^{\infty} \frac{(1)^n}{n!} \frac{z^n}{n!} \]

\[ \frac{e^{-z} - 1 + z}{z^2} = \frac{1}{2} - \frac{z}{3!} + \frac{z^2}{4!} - \ldots = \sum_{m=0}^{\infty} \frac{(-1)^m z^m}{(m+2)!} \]

This formula is valid for \( z \neq 0 \). But the series on the right converges on the whole complex plane, so it gives us an entire function \( g(z) \). Moreover, \( g(0) = \frac{1}{2} = f(0) \), so \( g(z) = f(z) \) everywhere.

4. Since \( f(z) = e^{iz} \) is an entire function and the circle \( \{ |z| = 2\pi \frac{3}{2} \} \) goes around the point \(-\pi\), the Cauchy Formula is applicable:

\[ \oint \frac{e^{iz}}{z + \pi} \, dz = 2\pi i \cdot e^{i(-\pi)} = -2\pi i \]