Show that the function defined by means of the equations

\[ f(z) = \begin{cases} 
(1 - \cos z) / z^2 & \text{when } z \neq 0 \\
\frac{1}{2} & \text{when } z = 0
\end{cases} \]

is entire.

Solution: \( \cos z \) is represented by convergent series:

\[ \cos z = 1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \frac{z^6}{6!} \ldots \]

for \( z \neq 0 \)

\[ \frac{(1 - \cos z)}{z^2} = \frac{1}{2} - \frac{z^2}{4!} + \frac{z^4}{6!} - \ldots \quad (*) \]

So for \( z \neq 0 \) \( f(z) \) is represented by series \((*)\) \( \text{The value of} \)

By definition of \( f \), \( f(0) = \frac{1}{2} \) is equal to the series \((*)\) at 0.

So \( f(z) = \frac{1}{2} - \frac{z^2}{4!} + \frac{z^4}{6!} - \frac{z^6}{8!} \ldots \) is representation of \( f \)

by a series convergent in the whole plane.

\( f \) is entire.