Hw 15

Fig 8 Fix $x_0 \in [A, B]$. Exp maps vertical segment $\{x_0 + iy, 0 \le y \le \pi\}$ onto an ark $e^{x_0 + iy} = re^{i\theta}$, where $r = e^{x_0}$, $\theta \in [0, \pi]$

So the image of the rectangle is the upper half of the anulus with radii e^A and e^B.

Or, if we fix $y_0 \in [0, \pi]$, the image of a horizontal segment $\{x + iy_0, A \le x \le B\}$ is a radial segment ~~e^{x+iy_0}~~ $\{re^{i\theta}$, where $\theta = y_0$, $e^A \le r \le e^B$. $\}$

Hw 15

Fig 9.

The ray $DE = \{-\frac{\pi}{2} + iy, y \geq 0\}$ is mapped onto the ray $[-\infty, -1]$, since

$$\sin\left(-\frac{\pi}{2} + iy\right) = \frac{1}{2i}\left[e^{i(-\frac{\pi}{2} + iy)} - e^{-i(-\frac{\pi}{2} + iy)}\right] =$$

$= -\frac{1}{2}\left[e^{-y} + e^{y}\right]$. In particular $-\frac{\pi}{2}$ is mapped to -1

$(\sin(-\frac{\pi}{2}) = -1!)$

Similarly the ray $\{\frac{\pi}{2} + iy, y \geq 0\}$ is mapped onto the ray $[1, \infty)$, and $[-\frac{\pi}{2}, \frac{\pi}{2}]$ is mapped by $\sin z$ onto $[-1, 1]$.

So the boundary of the first domain is mapped onto the boundary of the second. By argument principle, the first domain is mapped onto the second.

HW 15

Fig 19 $\quad z \mapsto \frac{z-1}{z+1}$ maps the upper half plane onto itself, and Log maps the upper half plane onto the strip $\{0 \le y \le \pi.\}$