Let $Z = \frac{z-1}{z+1}$. Show that when the principal branch of the square root is used, the composite function

$$w' = Z^{1/2} = \left(\frac{z-1}{z+1}\right)^{1/2}$$

maps the $z$-plane, except for the segment $-1 \leq x \leq 1$ of the $x$-axis, onto the right half plane $u > 0$.

**Solution**

1) First we prove that $Z = \frac{z-1}{z+1}$ maps the segment $-1 \leq x \leq 1$ of the $x$-axis onto the segment $X \leq 0$ of the $X$-axis. Clearly it maps real axis to real axis, and $\forall x \in [-1, 1]$ $X = \frac{x-1}{x+1} \leq 0$.

For $x = 1$, $X = 0$, for $x = -1$, $X = -\infty$. By Intermediate Value Theorem, $X$ takes on all values in between $-\infty$ and 0. So $Z$ maps the segment of the $x$-axis $[-1, 1]$ onto the segment $X \leq 0$.

2) Now we prove that $Z = \frac{z-1}{z+1}$ maps the $\{Z$-plane \ the segment $[-1, 1]\}$ of the $x$-axis's $Y$ onto the $\{Z$-plane \ the segment $X \leq 0$ of $X$-axis's $Y$.

It follows from Example 2 Section 102 that $\frac{z-1}{z+1}$ maps the $x$-axis onto the $X$-axis and the half planes $y > 0$ and $y < 0$ onto the half planes $y > 0$ and $y < 0$ respectively.
3) The principal branch of $Z^{1/2}$ maps the $Z$-plane \( \backslash \) the ray $X \leq 0$ \( \cup \) onto the right half plane.

So the composite function \( \left( \frac{Z-1}{Z+1} \right)^{1/2} \) maps $Z$-plane except for the segment $[-1,1]$ of the $x$-axis onto the right half plane.