Problem Set #9

1. doCarmo, Chapter 5, # 2, 3, 5, 7
2. Let H^2 be the upper half-plane with the Poincare metric. For any point $p \in H^2$ and any $r > 0$, let $B_p(r) = \{x \in H^2 \mid d(p,x) \leq r\}$.
   (a) Prove that $\partial B_p(r)$ "looks" like a sphere, that is, there exist real numbers, a,b, and R such that
   $$\partial B_p(r) = \{(x,y) \in H^2 \mid (x-a)^2 + (y-b)^2 = R^2\}$$
   (although $(a,b) \neq p$ and $R \neq r$).
   (b) Prove that Vol $(B_p(r))$ is an exponential function of r.
3. Let (M,g) and (N,h) be Riemannian manifolds. We define a Riemannian metric on $X = M \times N$ as follows. Let Let $\pi_1 : X \to M$ and $\pi_2 : X \to N$ be the natrual projections. Then for any $v, w \in T_{(p,q)}X$, define
   $$\langle v, w \rangle = g(d\pi_1(v), d\pi_1(w)) + h(d\pi_2(v), d\pi_2(w))$$
   (this is called the *product metric* on $M \times N$). Compute the curvature of X in terms of the curvatures of M and N.