Problem Set #8

1. doCarmo, Chapter 4, # 3, 4, 8, 9, 10
2. In proving Gauss's equation for surfaces, we computed the third derivatives, $\mathbf{x}_{ijk}$, of the parameterization. Setting mixed partial derivatives equal to each other, we found that

$$L_{ij}L_{lq} - L_{il}L_{jq} = \sum_m \left(\Gamma^m_{il,j} - \Gamma^m_{ij,l} + \sum_k (\Gamma^k_{il}\Gamma^m_{kj} - \Gamma^k_{ij}\Gamma^m_{kl}) \right) g_{mq}$$

   Prove that the right hand side is equal to $\langle R(\frac{\partial}{\partial x^i}, \frac{\partial}{\partial x^j})\frac{\partial}{\partial x^l}, \frac{\partial}{\partial x^q}\rangle$.
3. Let G be a Lie group with a bi-invariant metric. Prove that for any $p \in G$ and any section $\sigma \subset T_pG$

$$K(\sigma) = \frac{1}{4}\|[X, Y]\|^2$$

   where X and Y are orthonormal left–invariant vector fields such that $\{X(p), Y(p)\}$ span σ.