Christoffel Symbol Formulas

Many formulas involving the Christoffel Symbols are long and messy. It is not a good use of your time to memorize all of the indices. Therefore, you will be provided with the following formulas on the exam. Do not read too much into this. It does not mean that you will need all (or any) of these formulas. However, they will be available if you want them.

\[
\Gamma^1_{11} E + \Gamma^2_{11} F = \frac{1}{2} E_1 \quad \Gamma^1_{12} E + \Gamma^2_{12} F = \frac{1}{2} E_2 \quad \Gamma^1_{22} E + \Gamma^2_{22} F = F_2 - \frac{1}{2} G_1 \\
\Gamma^1_{11} F + \Gamma^2_{11} G = F_1 - \frac{1}{2} E_2 \quad \Gamma^1_{12} F + \Gamma^2_{12} G = \frac{1}{2} G_1 \quad \Gamma^1_{22} F + \Gamma^2_{22} G = \frac{1}{2} G_2
\]

If \( v(t) = a(u_1(t), u_2(t))x_1 + b(u_1(t), u_2(t))x_2 \), then

\[
\frac{Dv}{dt} = (a' + \Gamma^1_{11} au_1' + \Gamma^1_{12} au_2' + \Gamma^2_{11} bu_1' + \Gamma^2_{12} bu_2')x_1 + (b' + \Gamma^1_{11} au_1' + \Gamma^2_{12} au_2' + \Gamma^2_{11} bu_1' + \Gamma^2_{22} bu_2')x_2
\]

Gauss Equation:

\[
(\Gamma^1_{12})_1 - (\Gamma^2_{11})_2 + \Gamma^1_{12} \Gamma^2_{11} + \Gamma^1_{22} \Gamma^2_{12} - \Gamma^3_{11} \Gamma^2_{22} - \Gamma^1_{11} \Gamma^2_{12} = -EK
\]

Codazzi Equations:

\[
e_2 - f_1 = e \Gamma^1_{12} + f(\Gamma^2_{12} - \Gamma^1_{11}) - g \Gamma^2_{11} \\
f_2 - g_1 = e \Gamma^1_{22} + f(\Gamma^2_{22} - \Gamma^1_{21}) - g \Gamma^2_{21}
\]