Problem Set #9

due Monday, March 29, 2004

Warning: This is not the final draft of the assignment. More problems will be added in the course of the week.

1. doCarmo, section 3.3, # 5, 6, 16
2. Let V be a vector space with an inner product, $\langle \cdot, \cdot \rangle$, and let $\{v_1, v_2, \ldots, v_n\}$ be a (not necessarily orthonormal) basis of V. Prove that for any vector, $v = \sum c_i v_i$, the coefficients c_i can be computed by

$$\begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{pmatrix} = (\mathfrak{g}^T)^{-1} \cdot \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$$

   where $\mathfrak{g}$ is the matrix representing the inner product (ie. $\mathfrak{g}_{ij} = \langle v_i, v_j \rangle$) and $b_i = \langle v, v_i \rangle$.
3. Let S be a tubular surface about a curve α (see doCarmo, section 2.4, problem 10). Compute the second fundamental form, principal, Gaussian, and mean curvatures of S, in terms of the curvature and torsion of α.