Problem Set #5

due Monday, March 1, 2004

Note: The definition of a regular surface that I gave in class is not the same as the one in the book. doCarmo assumes that his coordinate patches are homeomorphisms. It is a theorem (Proposition 4 in section 2.2) that this is equivalent to the coordinate patches being 1-1. You may use whichever definition is more convenient.

1. doCarmo, section 2.2, # 2, 4, 10
2. Let $c(s) = (x(s), 0, z(s))$ be a simple regular curve in the xz–plane with $x(s) > 0$ for all s. Let S be the set of points formed by rotating $c(s)$ about the z–axis.
   (a) Show that $F(s, \theta) = (x(s)\cos\theta, x(s)\sin\theta, z(s))$ is a coordinate patch for some open set in the $s\theta$–plane.
   (b) Prove that the set S is a regular surface (called a *surface of revolution*).
   (c) Prove that S is still a regular surface if $c(s)$ is a simple closed curve.