Problem Set #5

due Monday, March 1, 2004

Note: The definition of a regular surface that I gave in class is not the same as the one in the book. doCarmo assumes that his coordinate patches are homeomorphisms. It is a theorem (Proposition 4 in section 2.2) that this is equivalent to the coordinate patches being 1-1. You may use whichever definition is more convenient.

1. doCarmo, section 2.2, # 2, 4, 10

2. Let \( c(s) = (x(s), 0, z(s)) \) be a simple regular curve in the \( xz \)-plane with \( x(s) > 0 \) for all \( s \). Let \( S \) be the set of points formed by rotating \( c(s) \) about the \( z \)-axis.

(a) Show that \( F(s, \theta) = (x(s) \cos \theta, x(s) \sin \theta, z(s)) \) is a coordinate patch for some open set in the \( s\theta \)-plane.

(b) Prove that the set \( S \) is a regular surface (called a surface of revolution).

(c) Prove that \( S \) is still a regular surface if \( c(s) \) is a simple closed curve.