Problem Set #3

due Monday, February 16, 2004

1. doCarmo, section 1.5, # 2,9,14
2. Let $\alpha(s)$ be a regular curve, parameterized by arclength, such that $\kappa(s) \neq 0$ and $\tau(s) \neq 0$ for all s.
   (a) Prove that if α lies on the sphere of radius r, centered at p, then
   $$\frac{\tau}{\kappa} = \left(\frac{\kappa'}{\tau\kappa^2}\right)'$$
   (b) Prove that the center of the sphere, p, satisfies
   $$p = \alpha(s) + \frac{1}{\kappa(s)}N(s) + \frac{\kappa'(s)}{\tau(s)\kappa^2(s)}B(s)$$
   for all s.
   (c) Prove the converse of part (a).
3. Find a minimal set of first–order, linear differential equations which are equivalent to the Frenet–Serret equations for a curve in $\mathbb{R}^3$. (Hint: You will need at least three equations.)