Problem Set #3

due Monday, February 16, 2004

1. doCarmo, section 1.5, # 2, 9, 14

2. Let \( \alpha(s) \) be a regular curve, parameterized by arclength, such that \( \kappa(s) \neq 0 \) and \( \tau(s) \neq 0 \) for all \( s \).
   (a) Prove that if \( \alpha \) lies on the sphere of radius \( r \), centered at \( p \), then
   \[
   \frac{\tau}{\kappa} = \left( \frac{\kappa'}{\tau \kappa^2} \right)'
   \]
   (b) Prove that the center of the sphere, \( p \), satisfies
   \[
   p = \alpha(s) + \frac{1}{\kappa(s)} N(s) + \frac{\kappa'(s)}{\tau(s) \kappa^2(s)} B(s)
   \]
   for all \( s \).
   (c) Prove the converse of part (a).

3. Find a minimal set of first-order, linear differential equations which are equivalent to the Frenet–Serret equations for a curve in \( \mathbb{R}^3 \). (Hint: You will need at least three equations.)