Problem Set #2

due Monday, February 9, 2004

1. Prove the following version of the Mean Value Theorem for vector valued functions:

   Let $\alpha : [a,b] \to \mathbb{R}^3$ be a regular, smooth curve. Then for any $\epsilon > 0$ there exists a $\delta > 0$ such that if $|c-d| < \delta$ then there exists a $\tau \in [c,d]$ for which

   $$\big|\alpha(c) - \alpha(d) - \alpha'(\tau)|c-d|\big| < \epsilon\big|c-d\big|$$

2. Let $\alpha(s)$ be a regular curve parameterized by arclength and let $R : \mathbb{R}^3 \to \mathbb{R}^3$ be a rigid motion. If $\beta(s) = R \circ \alpha(s)$, then
   (a) Compute $\beta'(s)$ is terms of $\alpha'(s)$.
   (b) Prove that $\beta(s)$ is a regular curve, parameterized by arclength, and that $\kappa_\beta(s) = \kappa_\alpha(s)$.

3. Compute the curvature of the logarithmic spiral, $\alpha(t) = (e^{-t}\cos t, e^{-t}\sin t, 0)$.