MAT 141
Problem Set #12
due in recitation on November 23 or 24, 2004

1. Prove that if \( f(x) \) is bounded on \([a, b]\) and discontinuous at only finitely many points of \([a, b]\), then \( f(x) \) is integrable on \([a, b]\).

2. Assume that \( f(x) \) is continuous on \([a, b]\). You are asked to defend the statement \( \lim_{x \to p} f(x) = f(p) \). Given a challenge, \( \epsilon \), describe a strategy for finding your response, \( \delta \), that does not depend on \( p \). (Hint: You will need to use the small span theorem.)

3. Consider the function

\[
g(x) = \begin{cases} 
x^2 & \text{if } x \text{ is rational} \\
0 & \text{if } x \text{ is irrational} 
\end{cases}
\]

Prove that \( g(x) \) is differentiable at \( x = 0 \). Compute \( g'(0) \).