MATH 132 - LECTURE 1
JULY 24, 2017

Homework 3

Due on Monday, August 7.

Exercise 1 Compute the volumes of the following solids:

(a) The solid obtained by rotating the region bounded by the curves $y = x^3$, $y = \sqrt{x}$ and $x \geq 0$, about the y-axis.

(b) The solid obtained by rotating the region bounded by the curves $x = 2y - y^2$ and $x = 0$, about the $y - axis$.

(c) The solid obtained by rotating the region bounded by the curves $x = 1 + y^2$, $x = 0$, $y = 1$ and $y = 2$, about the $x - axis$. (Hint: Use cylindrical shells).

Exercise 2 A bowl is shaped like a hemisphere with radius 30cm. A very heavy ball with diameter 10cm is placed in the bowl. Water is poured into the bowl to a depth of 20 cm, completely submerging the ball. Find the volume of water in the bowl.

Exercise 3 Find the lengths of the following curves:

(a) The curve given by the equation $y^2 = 4(x + 4)^3$ and the inequalities $0 \leq x \leq 2$, $y > 0$.

(b) The astroid: $x(t) = 2[\cos(t)]^3$, $y(t) = 2[\sin(t)]^3$, $0 \leq t \leq 2\pi$.

(c) The infinite spiral: $x(t) = e^{-t}\cos(t)$, $y(t) = e^{-t}\sin(t)$, $0 \leq t \leq +\infty$.

Exercise 4 A CAT scan produces equally spaces cross-sectional images of a human organ and provide approximate information otherwise only obtainable through surgery.

Suppose that research suggests that the length of a patient's liver is approximately 15 cm long. A CAT scan of this organ shows cross-sections whose areas have been approximated by a computer, and measure (in square centimeters): 0, 18, 58, 79, 94, 106, 117, 128, 63, 39 and 0. Use Simpson's rule to estimate the volume of this patient's liver.

Exercise 5 Match the direction fields to the differential equations. Explain your reasoning.

(a)

$$\frac{dy}{dx} = x$$

(b)

$$\frac{dy}{dx} = y(y - 2)$$

(c)

$$\frac{dy}{dx} = y(x - 1)$$

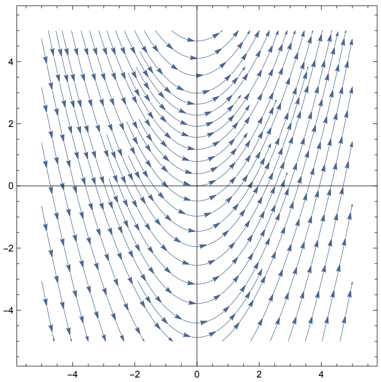

Figure 1:

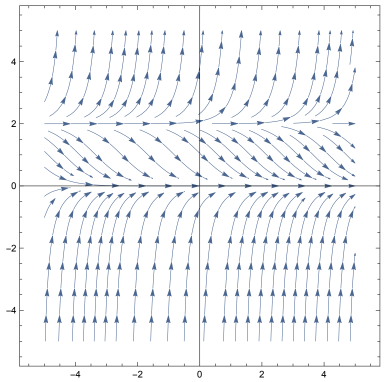

Figure 2:

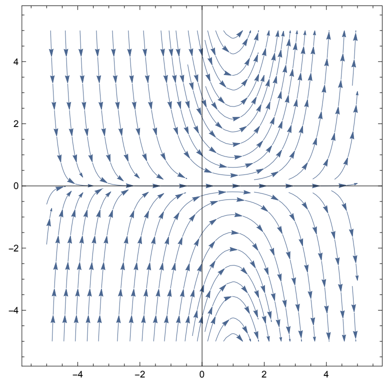

Figure 3:

Exercise 6 Sketch the direction field of the differential equation

$$(y^2+1)\frac{dy}{dx}+(1-y^2)=0.$$

Without solving the differential equation, answer the following:

(a) What are the equilibrium solutions?

(b) What is the long-term behavior of the solutions with initial value $y(0)=0$? Explain your reasoning.

(c) What is the long-term behavior of the solutions with initial value $y(0)=2$? Explain your reasoning.

(d) What is the long-term behavior of the solutions with initial value $y(0)=-2$? Explain your reasoning.

Exercise 6 Solve the following equations

(a) (5 points)

$$\frac{dy}{dx}=\frac{\log(x)}{xy} \qquad y(1)=2.$$

(b) (5 points)

$$y(x)=2+\int_1^x \frac{1}{ty(t)}dt$$

Exercise 7 Use Euler's method with step size 0.2 to estimate y(1), where $y(x)$ is the solution to the initial-value problem

$$\frac{dy}{dx} - y^2 - x = 0, \qquad y(0) = 0.$$

Exercise 8 Suppose a population $P(t)$ satisfies the logistic equation

$$\frac{dP}{dt} = 0.4P - 0.001P^2,$$

where t is measured in years. The initial population $P(0) = 0$.

(a) What is the carrying capacity of the environment?

(b) What is the initial growth rate $P'(0)$?

(c) When will the population reach 50% of the carrying capacity?

Exercise 9 Find the orthogonal trajectories to the family of ellipses given by

$$x^2 + 2y^2 = k^2$$

Exercise 10 Find the solution for the initial-value problem

$$y'' - 2y' + 4y = 0, \qquad y(0) = 0, y'(0) = 1.$$