MAT 205 SPRING 2000
FINAL EXAM

Name:

SSN:

THERE ARE TEN PROBLEMS. SHOW YOUR WORK!

1. Let $z = x \cos y$, $x = t^4$, $y = e^{2t}$. Find $\frac{dz}{dt}$.

2. Find the equation of the plane containing the points $(0, 2, 0)$, $(1, 1, 0)$, and $(-1, 0, 1)$.

3. Find the derivative of the function $f(x, y) = e^{2x+y}$ in the direction of the unit vector $\frac{\sqrt{3}}{2}\mathbf{i} + \frac{-1}{2}\mathbf{j}$ at the point $(1, -1)$.

4. The velocity vector of a particle is given by $\mathbf{v(t)} = 3t^2\mathbf{i} + e^{2t}\mathbf{j} + t\mathbf{k}$. Find $\mathbf{r(1)}$, the position vector of the particle when $t = 1$, given that $\mathbf{r(0)} = \mathbf{i} + \mathbf{j}$.

5. Compute

$$\int\int_D x \cos x^2 dA$$

where D is bounded by $y = 0$, $y = \pi$, $y = x^2$, and $x \geq 0$.

6. Calculate

$$\int\int_S \mathbf{F} \cdot \mathbf{dS}$$

where $\mathbf{F}(x, y, z) = ye^{z^2}\mathbf{i} + y^2\mathbf{j} + e^{xy}\mathbf{k}$ and S is the surface of the solid bounded by the cylinder $x^2 + y^2 = 9$ and the planes $z = 0$ and $z = 3$ given the outward orientation.

7. Calculate

$$\int\int_S curl\mathbf{F} \cdot \mathbf{dS}$$

where $\mathbf{F}(x, y, z) = yz\mathbf{i} + xz\mathbf{j} + x^2y^2\mathbf{k}$ and S is the part of the paraboloid $z = x^2 + y^2$, oriented upward, that lies inside the cylinder $x^2 + y^2 = 4$.

You will need to use the identity $\int \cos^2 t dt = \frac{1}{2}t + \frac{1}{4}\sin 2t + C$

8. Evaluate

$$\int_C xy\,dx + y^5\,dy$$

where C is the triangle with vertices $(0,0)$, $(2,0)$, and $(2,1)$ positively oriented.

9. Let $\mathbf{F} = (x + y^2)\mathbf{i} + (2xy + y^2)\mathbf{j}$.

1. Is $\mathbf{F}$ conservative?

2. Find a function $f(x, y)$ such that $\nabla f = \mathbf{F}$.

3. Find $\int_C \mathbf{F} \cdot \mathbf{dr}$ where C is any curve starting at $(0, 0)$ and ending at $(1, 1)$.

10. Evaluate

$$\int_C 3y\, ds$$

where C is the curve paramatrized by $x = t$, $y = t^3$, $0 \leq t \leq 1$.