MAT 127 FALL 2007
MIDTERM 2

!!! WRITE YOUR NAME, STUDENT ID AND LECTURE N. BELOW !!!

NAME : ID :

LECTURE N.

THERE ARE **5 MULTIPLE-CHOICE** PROBLEMS (20pts each)
and
4 PARTIAL CREDIT PROBLEMS (40pts each)

SHOW YOUR WORK!!!

DO NOT TEAR-OFF ANY PAGE

NO CALCULATORS NO PHONES

ON YOUR DESK: **ONLY** test, pen/pencil, eraser.

1. Find the interval of convergence for the power series

$$\sum_{n=1}^{\infty} (-1)^n \frac{(x-3)^n}{n2^n}.$$

A. $(-2, 2]$
B. $(1, 5]$
C. $[1, 5]$
D. $(-\infty, \infty)$
E. converges only for $x = 3$
F. $(-2, 2)$
B
Apply ratio test:

$$\lim_{n\to\infty} \frac{|x-3|^{n+1}}{(n+1)2^{n+1}} \frac{n2^n}{|x-3|^n} = \lim_{n\to\infty} \frac{|x-3|}{2} < 1$$

for

$$|x-3| < 2.$$

Endpoints:
for $x = 5$, get the alternating harmonic series: convergent.
for $x = -1$, get the harmonic series: divergent.

2. Identify the Maclaurin series for

$$\int \frac{x}{1+x^3}dx$$

A. $\sum_{n=0}^{\infty} \binom{3}{n} x^{n+1} + C$

B. $\sum_{n=0}^{\infty} \frac{x^{3n+2}}{3n+2} + C$

C. $\sum_{n=0}^{\infty} \frac{x^{3n}}{n!} + C$

D. $\sum_{n=0}^{\infty} (-1)^n \frac{x^{3n+1}}{3n+1} + C$

E. $\sum_{n=0}^{\infty} (-1)^n \frac{x^{3n+2}}{3n+2} + C$

F. $\sum_{n=0}^{\infty} (-1)^n \frac{x^{n+2}}{n+2} + C$

E

$$\frac{1}{1+x^3} = \sum_0^{\infty} (-1)^n x^{3n}.$$

$$x\frac{1}{1+x^3} = \sum_0^{\infty} (-1)^n x^{3n+1}$$

Integrate term by term and add a constant.

3. Find the sum of the series

$$1+2+\frac{4}{2!}+\frac{8}{3!}+\frac{16}{4!}+\dots$$

(assuming that the given pattern continues).

A. $\sin(2)$
B. $\tan^{-1}(-2)$
C. e^2
D. e^{-2}
E. $\frac{\pi}{3}$
F. This series diverges.

C
It is the MacLaurin series for

$$e^x=\sum_0^\infty \frac{1}{n!}x^n$$

evaluated at $x=2$.

4. The Maclaurin series for

$$\frac{1}{\sqrt{1+x}}$$

is $1 + \sum_{n=1}^{\infty} c_n x^n$. Identify c_n

A. $\frac{(-1/2)(-3/2)...(-1/2-n+1)}{n!}$

B. $\frac{(1/2)(-1/2)...(1/2-n+1)}{n!}$

C. $\frac{(1/2)(3/2)...(1/2+n-1)}{n!}$

D. $\frac{1}{n!}$

E. $(-1/2)(-3/2)\dots(-1/2-n+1)$

F. $(\frac{1}{2})^n$

A

$$\frac{1}{\sqrt{1+x}} = (1+x)^{-1/2}.$$

Apply the binomial theorem for $k = \frac{-1}{2}$.

5. Consider $f(x) = \sin(2x)$. If $|R_3(x)|$ is the error of the 3rd degree Taylor polynomial approximation centered at zero, then

A. $|R_3(x)| \leq \frac{8}{3!}|x|^3$

B. $|R_3(x)| \leq \frac{1}{3!}|x|^3$

C. $|R_3(x)| \leq \frac{16}{4!}|x|^4$

D. $|R_3(x)| \leq \frac{1}{4!}|x|^4$

E. $|R_3(x)| \leq \frac{1}{4!}$

F. $|R_3(x)| \leq \frac{8}{3!}$

C

Use Taylor's inequality

$$|R_3(x)| \leq \frac{M}{4!}|x|^4$$

with $M \geq |f^{(4)}(x)| = |-16\cos 2x|$.

$M = 16$ does it.

6. Use the 4th degree Taylor polynomial of $\cos(x)$ centered at 0 to approximate $\cos(1)$. Estimate the error in making this approximation.

$$T_4(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}.$$

$$T_4(1) = 1 - \frac{1}{2} + \frac{1}{4!} = \frac{13}{24}.$$

Solution 1: The series for $\cos 1$ is alternating. The error is $\leq$ the absolute value of the next term, that is:

$$|error| \leq \frac{1}{6!}.$$

Solution 2: Use Taylor's Estimate: with $M = 1 \geq cos^{(5)}(x)| = |\sin x|$ and interval $|x| \leq 1$:

$$|error| \leq \frac{1}{5!}.$$

Note: can use the fact that $T_4(x) = T_5(x)$, use Taylor's estimate and get

$$|error| \leq \frac{1}{6!}.$$

7. Find the Taylor series for $f(x) = \ln x$ centered at 1 and find its radius of convergence.

$$f^{(n)}(x) = (-1)^{n-1}(n-1)!x^{-n}.$$

The Taylor series at 1 is

$$= \sum \frac{(-1)^{n-1}}{n}(x-1)^n.$$

The radius of convergence is $R = 1$ by the ratio test.

8. Using the Maclaurin series of

$$\frac{1}{1-x}$$

find the Maclaurin series of the function

$$f(x) = \frac{1}{1+4x^2}.$$

Then find its radius of convergence.

$$\frac{1}{1-x} = \sum_0^\infty x^n$$

with radius $R = 1$.

Write

$$\frac{1}{1+4x^2} = \frac{1}{1-(-4x^2)} = \sum_0^\infty (-4x^2)^n =$$

$$\sum_0^\infty (-4)^n x^{2n}.$$

For the radius of convergence:

$$|4x^2| < 1,$$

or

$$|x| < \frac{1}{2}.$$

The radius is $R = \frac{1}{2}$.

9. a) Use binomial series to expand

$$\frac{1}{(1+x)^2}$$

as a power series.

b) use part a) to find the sum of the series

$$\sum_{n=0}^{\infty}(-1)^n\frac{n+1}{3^n}$$

a)

$$(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + 5x^4 \ldots = \sum_{0}^{\infty}(-1)^n(n+1)x^n$$

b) it is the series for $x = \frac{1}{3}$ which is in the interval of convergence. So the series equals

$$(1+\frac{1}{3})^{-2} = \frac{9}{16}.$$