MAT 127 FALL 2007
MIDTERM I

!!! WRITE YOUR NAME, STUDENT ID AND LECTURE N. BELOW !!!

NAME : ID :

LECTURE N.

THERE ARE 10 MULTIPLE-CHOICE PROBLEMS. THEY HAVE THE INDICATED VALUE.

DO NOT TEAR-OFF ANY PAGE

NO CALCULATORS NO PHONES

ON YOUR DESK: ONLY test, pen, pencil eraser.

1		25pts
2		25pts
3		25pts
4		25pts
5		25pts
6		25pts
7		25pts
8		25pts
9		25pts
10		25pts
Total		250pts

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1. Let a_n be a convergent sequence such that $\lim a_n = L$.
Define a new sequence $b_n = 2 - a_n$. Then $\lim b_n =$ a

A. L
B. $L - 2$
C. $2 - L$
D. $2L$
E. $-2L$
F. L^2

C The limit of $2 - a_n$ is $2 - L$.

2. The sequence $a_n = e^n - e^{-n}$ is

A. Bounded and monotonic.
B. Bounded.
C. Monotonic decreasing.
D. Bounded and monotonic increasing.
E. Bounded below.
F. Bounded below and monotonic decreasing.

E The sequence is positive, hence bounded below.
It is not decreasing. It is not bounded.

3. Find the sum of the series

$$-\frac{2}{9}+\frac{2}{27}-\frac{2}{81}+\frac{2}{243}\cdots$$

(assuming that the given pattern continues).

A. $\frac{1}{6}$
B. $-\frac{1}{6}$
C. $\frac{3}{2}$
D. $-\frac{3}{2}$
E. $-\frac{2}{11}$
F. $-\frac{5}{3}$

$\mathbf{B}=\frac{2}{3}\sum_1^\infty(-\frac{1}{3})^n=\frac{2}{3}(\frac{1}{1+\frac{1}{3}}-1)=-\frac{1}{6}$

4. Find the sum of the series

$$\sum_{n=3}^{\infty} \frac{4}{n^2 - 4}.$$

(Hint: write the given fraction as a difference of fractions.)

A. The sum diverges.
B. $\frac{4}{3}$
C. $\frac{17}{25}$
D. $\frac{25}{7}$
E. $\frac{13}{22}$
F. $\frac{25}{12}$

F $\frac{4}{n^2-4} = \frac{1}{n-2} - \frac{1}{n+2}$
It is a telescopic sum:

$$(\frac{1}{1} - \frac{1}{5}) + (\frac{1}{2} - \frac{1}{6}) + (\frac{1}{3} - \frac{1}{7}) + (\frac{1}{4} - \frac{1}{8}) + (\frac{1}{5} - \frac{1}{9}) + \ldots$$

All terms cancel except: $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{25}{12}$

5. Which of the following series is convergent but not absolutely convergent?

$$(A)\ \sum_{n=1}^{\infty}\frac{(-1)^n}{n^{3/2}} \qquad (B)\ \sum_{n=1}^{\infty}(-1)^{n-1}\frac{3^n}{n^4}$$

$$(C)\ \sum_{n=1}^{\infty}\frac{(-10)^n}{n!} \qquad (D)\ \sum_{n=1}^{\infty}\frac{n^2}{(-3)^n}$$

$$(E)\ \sum_{n=1}^{\infty}(-1)^n\frac{2n}{n^2+1} \qquad (F)\ \sum_{n=1}^{\infty}(-1)^{n-1}ne^{-n}$$

E is convergent by alternating series test, but the series of absoulte values is divergent by the limit comparison test with $\sum \frac{2}{n}$.

Just to clarify:

all the other ones converge absolutely by the ratio test, except for (B) that diverges.

6. Which of the following assumptions about the series $\sum a_n$ can tell us the series is convergent?

$$(A)\ \lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| = 4 \qquad (B)\ \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1$$

(C) $\lim_{n\to\infty} a_n = 0$
(D) $\sum_{n=1}^{\infty} a_n$ is an alternating series and $|a_{n+1}| \leq |a_n|$.
(E) There is a series $\sum_n b_n$ with positive terms, such that $|a_n| < b_n$ for all n and such that $\sum_{n=1}^{\infty} b_n$ is convergent.
(F) The sequence of partial sums s_n of the series is bounded above.

E By the comparison test for convergence.
Just to clarify:
(A) gives divergence by ratio test
(B) ratio test is inconclusive
(C) counterxample: harmonic series
(D) a_n must go to zero and that is not assumed in C
(F) a sequence can be bounded above, even bounded, and still not converge

7. The series

$$\sum_{n=2}^{\infty} \frac{n^3 - n^2 + n - 1}{n^5 - n^3 + n}.$$

A. Converges because a_n converges to zero.
B. Diverges by the test for divergence.
C. Diverges by the ratio test.
D. Converges by the limit comparison test.
E. Converges by the ratio test.
F. Diverges by the limit comparison test.

D compare with $\sum \frac{1}{n^2}$

8. Suppose a sequence $\{a_n\}_{n=1}^{\infty}$ satisfies

$$1 - a_{n+1} = q(a_n + a_{n-1} + a_{n-2} + \ldots + a_2 + a_1)$$

for some fixed nonzero constant q. Assume the series $\sum_{n=1}^{\infty} a_n$ converges and denote the sum of the series by s.
Find the value of s.
(Hint: the sum of the series $s = \lim_{n\to\infty} s_n$, where s_n is the sequence of partial sums.)

A. 1
B. 0
C. $\frac{1}{nq+1}$
D. $\frac{1-a_{n+1}}{q}$
E. $\frac{1}{q}$
F. q

E
$s_n = a_1 + \ldots + a_n$.
$a_{n+1} = 1 - qs_n$.
$s_n \to s$ and $a_n \to 0$.
Taking limits on both sides:

$$0 = 1 - qs.$$

So $s = 1/q$.

9. Consider the following series

$$(1)\sum_{n=1}^{\infty}\frac{(-1)^n}{n^4} \qquad (2)\sum_{n=1}^{\infty}\frac{1+\sin n}{n-5\ln n}$$

$$(3)\sum_{n=1}^{\infty}\frac{(-3)^{n-1}}{\sqrt{n}} \qquad (4)\sum_{n=1}^{\infty}\frac{\sqrt{2n}}{1+n^2}$$

$$(5)\sum_{n=1}^{\infty}\frac{1}{n(n+1)} \qquad (6)\sum_{n=1}^{\infty}\frac{x^n}{n!}\text{ for any number } x<0$$

Which of the above series are alternating series?
(A) (1) and (5)
(B) (1) and (6)
(C) (2) and (3)
(D) (1), (3) and (6)
(E) (1)
(F) (5)

D is correct, but so are B and E.
(2) (4) and (5) are positive.

10. The series $\sum_1^\infty e^{-2n}$ converges. Find n so that the remainder R_n of the series satisfies
$$\frac{e^{-10}}{2} \le R_n \le \frac{e^{-7}}{2}.$$

A 2
B. 3
C. 4
D. 5
E. 6
F. There is no such value of n.

C
By remainder integral estimate
$$\int_{n+1}^{\infty} e^{-2x} \le R_n \le \int_{n}^{\infty} e^{-2x}$$
Using the primitive $-\frac{1}{2}e^{-2x}$ we get
$$\frac{1}{2}e^{-2n-2} \le R_n \le \frac{1}{2}e^{-2n}.$$
$n = 4$ works because $e^{-8} \le e^{-7}$.