1. Exercises:

(1) Solve problems 3A, 3C, 3D, 3E(first half), 3F (especially if you are an algebraic geometer!).

(2) For any ring R, compute $H^*(\mathbb{RP}^n; R)$ by induction. (Hint: Show that $\mathbb{RP}^n$ can be obtained from $\mathbb{RP}^{n-1}$ by gluing in an n-cell attached in a particular way.)

Compute the cup product as well.

(3) (Note: all manifolds in this course are Hausdorff and paracompact.)

Show that if a smooth vector bundle has a global trivialization as a *topological* vector bundle then it has a global trivialization as a *smooth* vector bundle using the Steenrod approximation theorem:

Theorem 1.1. (Steenrod approximation theorem.) Let $\pi : V \to B$ be a smooth vector bundle and suppose that we have a fixed metric $|\cdot|$ on V. Then every continuous section s of π can be 'approximated' by a smooth section. More precisely, for every $\epsilon > 0$, there exists a smooth section s' of π so that $|s - s'| < \epsilon$.

(4) More generally, show that if two smooth vector bundles are isomorphic as topological vector bundles then they are isomorphic as smooth vector bundles.

(5) Prove the Steenrod approximation theorem assuming that the following Theorem is true:

Theorem 1.2. Let $f : B \to \mathbb{R}$ be a continuous function on a manifold B and let $U \subset B$ be an open set and $A \subset U$ a closed subset of U. Suppose that $f|_U$ is smooth. Then for all $\epsilon > 0$ there is a smooth function $g : B \to \mathbb{R}$ so that $g|_A = f|_A$ and so that $|f(x) - g(x)| < \epsilon$ for all $x \in B$.