

MAT 542 HOMEWORK 1

- (1) (Hatcher 4.2 8) Show that the suspension of an acyclic CW complex is contractible. (Recall that a CW complex is acyclic if all reduced homology groups vanish.)
- (2) Compute $\pi_3(SU(n))$ for each $n \in \mathbb{N}$ where $SU(n) \subset \mathbb{C}^{n \times n}$ is the special unitary group (I.e. the set of $n \times n$ matrices U satisfying $UU^* = U^*U = 1$ and $\det_{\mathbb{C}} U = 1$).
- (3) (Hatcher 4.2 7) Construct a CW complex with prescribed homotopy groups $\pi_k(X)$, $k \in \mathbb{N}$ and prescribed actions of $\pi_1(X)$ on the $\pi_k(X)$'s.
- (4) Using the quaternionic Hopf map and also the suspension map $\pi_{k-1}(S^3) \rightarrow \pi_k(S^4)$, show that

$$\pi_k(S^4) \cong \pi_k(S^7) \oplus \pi_{k-1}(S^3) \quad \forall k \in \mathbb{N}.$$