HOMEWORK 3

(1) Consider the set $X = \{0, 1, 2\}$ with the topology

$$\mathcal{T} := \{\emptyset, \{0\}, \{2\}, \{0, 2\}, \{0, 1, 2\}\}.$$

What are its singular homology groups?

Hint: Invent your own 'bracket' with similar properties to the one defined for star convex spaces.

(2) Calculate the singular homology groups of the Warsaw circle, which is the union of the following three sets:

$$\left\{(x, \sin(\frac{1}{x})) \ : \ -\frac{1}{2\pi} < x < \frac{1}{2\pi}, x \neq 0\right\} \cup \{(0, y) \ : \ -1 \leq y \leq 1\} \cup C \subset \mathbb{R}^2$$

where $C \subset \mathbb{R}^2$ is a curve joining $(-\frac{1}{2\pi}, 0)$ and $(\frac{1}{2\pi}, 0)$ which is disjoint from the other two sets.

(3) Classify all abelian groups A that fit in to the following short exact sequence:

$$0 \longrightarrow \mathbb{Z} \longrightarrow A \longrightarrow \mathbb{Z}/n\mathbb{Z} \longrightarrow 0 \tag{1}$$

where $n > 0$ is an integer (Hatcher Section 2.1 Exercise 14).

(4) Prove the *five lemma*. I.e. given a commutative diagram of abelian groups

$$\begin{array}{ccccccccc} A & \longrightarrow & B & \longrightarrow & C & \longrightarrow & D & \longrightarrow & E \\ \alpha\downarrow & & \beta\downarrow & & \gamma\downarrow & & \delta\downarrow & & \epsilon\downarrow \\ A' & \longrightarrow & B' & \longrightarrow & C' & \longrightarrow & D' & \longrightarrow & E' \end{array}$$

where the horizontal arrows are exact, β,δ are isomorphisms, α is surjective and ϵ is injective, show that γ is an isomorphism.