(1) Let $K = (\sigma_i)_{i \in I}$ be a simplicial complex. Show that the quotient topology on $|K|$ coming from the map

$$\sqcup_{i \in I} \sigma_i \twoheadrightarrow |K|$$

is identical to the usual topology on $|K|$.

(2) For any subset J, show that addition

$$E^J \times E^J \longrightarrow E^J, \quad (x, y) \longrightarrow x + y$$

and scalar multiplication

$$\mathbb{R} \times E^J \longrightarrow E^J, \quad (\lambda, x) \longrightarrow \lambda x$$

are continuous functions.

(3) Let

$$S^n := \{(x_0, \cdots, x_n) \in \mathbb{R}^{n+1} \ : \ \sum_{j=0}^{n} x_j^2\} \subset \mathbb{R}^{n+1}$$

be the unit sphere. Let $\mathbb{RP}^n := S^n / \sim$ where $\sim$ is the equivalence relation identifying x with $-x$. Is it true that $\mathbb{RP}^2$ is homeomorphic to a simplicial complex?

Optional: What about $\mathbb{RP}^n$?

(4) (Munkres Ch 1 ex 4,7)
- Let $\mathcal{S} \subset \mathcal{P}(\mathbb{N})$ be the abstract simplicial complex consisting of subsets of size ≤ 2 and where each subset of size 2 contains 0. Draw a geometric realization of such a simplicial complex and show its topology is not first countable.

(5) (Munkres Ch 1 ex 7) Show each locally finite simplicial complex is metrizable (*Hint*: use barycentric coordinates).

(6) Show that there is a finite simplicial complex in $\mathbb{R}^n$ so that the union of its simplices in $\mathbb{R}^n$ is the cube $[0, 1]^n \subset \mathbb{R}^n$.