

## HOMEWORK 1

**Due:** Thursday September 6th at 10:00am in Physics P-124

*Please write your solutions legibly; the TA may disregard solutions that are not readily readable. All solutions must be stapled (no paper clips) and have your name (first name first) and HW number in the upper-right corner of the first page.*

**Problem 1:** Is the function

$$f : [0, 1] \longrightarrow \mathbb{R}, \quad f(x) := \begin{cases} 1 & \text{if } x \in [\frac{1}{2^k}, \frac{1}{2^{k-1}}] \text{ for some odd } k \\ -1 & \text{if } x \in (\frac{1}{2^k}, \frac{1}{2^{k-1}}) \text{ for some even } k \\ 0 & \text{if } x = 0 \end{cases}$$

Riemann integrable? Justify your answer.

**Problem 2:** Let  $f, g : [0, 1] \longrightarrow \mathbb{R}$  be continuous functions. Define

$$F : [0, 1] \longrightarrow \mathbb{R}, \quad F(x) := \begin{cases} f(x) & \text{if } x \text{ is rational} \\ g(x) & \text{otherwise.} \end{cases}$$

Compute the upper and lower Riemann integral of  $F$  in terms of integrals involving  $f$  and  $g$ .

**Problem 3:** Let  $f : \mathbb{R} \longrightarrow \mathbb{R}$  be a differentiable function whose derivative  $f'$  is bounded and let  $A \subset \mathbb{R}$  be a null set. Show that  $f(A)$  is null.

**Problem 4:** Let  $A, B$  be a null sets. Is it true that

$$A + B := \{a + b : a \in A, b \in B\}$$

is null? If it is then prove it, otherwise give a counterexample.