HW 8

5.2 (2) $v(0,y,t)=T_0, \quad v(a,y,t)=T_1$

$$-k\frac{\partial v}{\partial y}(x,0,t)+h\cdot v(x,0,t)=hT_2$$

$$k\frac{\partial v}{\partial y}(x,b,t)+h\cdot v(x,b,t)=hT_2$$

$$v(x,y,0)=\frac{1}{c}\int_0^c f(x,y,z)\,dz$$

(6) $\dfrac{\partial^2 v}{\partial x^2}+\dfrac{\partial^2 v}{\partial y^2}=0, \quad v(x,0)=f_1(x), \quad v(x,b)=f_2(x)$

$v(0,y)=g_1(y), \quad v(a,y)=g_2(y)$

This problem can be solved by splitting it into 2 subproblems:

$$\begin{cases} \nabla^2 v_1 = 0 \\ v_1(x,0)=f_1(x) \\ v_1(x,b)=f_2(x) \\ v_1(0,y)=0=v_1(a,y) \end{cases} \qquad \begin{cases} \nabla^2 v_2=0 \\ v_2(x,0)=0=v_2(x,b) \\ v_2(0,y)=g_1(y) \\ v_2(a,y)=g_2(y) \end{cases}$$

and adding the solutions as $v=v_1+v_2$.
The subproblems can be solved by separation of variables (as in sec. 4.2).

(12) Frequencies are $\dfrac{\lambda_{mn}c}{2\pi}=\dfrac{c}{2}\sqrt{\dfrac{m^2}{a^2}+\dfrac{n^2}{b^2}}=: f_{mn}$

For $a=b$, $f_{mn}=\dfrac{c}{2a}\sqrt{m^2+n^2}$. There are different pairs that give same frequencies, like $(1,2)$, $(2,1)$.

5.4 (6) $$\begin{cases} \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial \phi}{\partial r}\right)+\dfrac{1}{r^2}\dfrac{\partial^2\phi}{\partial\theta^2}=-\lambda^2\phi \\ \dfrac{\partial\phi}{\partial r}(a,\theta)=0, \quad \phi(r,\theta+2\pi)=\phi(r,\theta), \quad \phi \text{ bounded as } r\to 0 \end{cases}$$

$$\phi(r,\theta) = R(r)\cdot Q(\theta) \Longrightarrow \begin{cases} \dfrac{(rR')'}{rR} + \dfrac{Q''}{r^2 Q} = -\lambda^2 \\ R'(a)\,Q(\theta) = 0 \Rightarrow R'(a) = 0 \\ Q(\theta) = Q(\theta + 2\pi) \\ R(r) \text{ bounded as } r \to 0 \end{cases}$$

$\boxed{5.5}$ 10 General solution is $R(r) = A J_0(\lambda r) + B Y_0(\lambda r)$ (Bessel eq. with $\mu = 0$).

$\phi(0)$ bounded $\Rightarrow B = 0$. So $R(r) = A J_0(\lambda r)$.

$$\frac{\partial \phi}{\partial r}(a) = 0 \Rightarrow \frac{d}{dr}\left(J_0(\lambda r)\right)\Big|_{r=a} = -\lambda J_1(\lambda a) = 0 \Rightarrow$$

$$\Rightarrow \lambda_m = \frac{\alpha_{1n}}{a}, \quad \{\alpha_{1n}\} \text{ zeroes of } J_1.$$

Eigenfunctions are $\{J_0(\lambda_m r)\}_{m=1}^{\infty}$ with $\lambda_m = \dfrac{\alpha_{1n}}{a}$.