The second midterm is scheduled for Wednesday April 6th and it covers sections 2.1, 2.2, 2.3, 3.1, 3.2, 3.3 and 3.5 of the textbook. There will be two review sessions on Wednesday March 30th and Monday April 4th. You do not need to submit this practice midterm as an assignment homework, but you should know how to solve all the problems anyways. For further problems and exercises you can consult either the book or past midterms of previous MAT303 classes (see the link http://www.math.stonybrook.edu/mathematics-departmentcourse-web-pages).

**Exercise 1:** Consider an animal population $P(t)$ with constant death rate $\delta = 1/100$ and with birth rate $\beta$ proportional to $P$. Suppose that $P(0) = 200$ and $P'(0) = 2$. When does explosion occur?

**Exercise 2:** Consider the following population (autonomous) model:

$$\frac{dP}{dt} = P^2 - 3P + 2.$$ 

Find the equilibrium solutions and classify them as stable or unstable. Sketch then two solutions curves for the initial conditions $P(0) = 1.9$ and $P(0) = 2.1$. Finally Solve the differential equation with the initial condition $P(0) = 3$ and find at what time the population explodes to infinity.

**Exercise 3:** Consider the following differential equation:

$$\frac{dx}{dt} = (x + 2)(x - 2)^2.$$ 

Find the equilibrium solutions, classify them as stable or unstable, and sketch the solution curves corresponding to the initial conditions $x(0) = x_0$, where $x_0$ can be either a negative or a positive real number.

**Exercise 4:** The differential equation

$$\frac{dx}{dt} = \frac{1}{100}x(x - 5) + s$$

models a logistic population with stocking at rate $s$. Determine the bifurcation point and the equilibrium solutions as $s$ varies. Finally classify the equilibrium solutions as stable or unstable, and sketch the solution curves for the value $s = 0$.

**Exercise 5:** Suppose that a body moves through a resisting medium with resistance proportional to $v^\alpha$, so that

$$\frac{dv}{dt} = -kv^\alpha, \quad v(0) = v_0, \quad k = \text{positive constant}.$$
(a). If \( \alpha = 1 \), we have seen in a previous homework that the velocity function is \( v(t) = v_0 e^{-kt} \) with a position function \( x(t) = x_0 + \left( \frac{v_0}{k} \right) (1 - e^{-kt}) \). Moreover we have seen that the body travels only a finite distance.

(b). Find the velocity and position functions for \( \alpha = \frac{3}{2} \) and \( \alpha = 3 \). In which cases does the body travel a finite distance?

**Exercise 6:** Find the general solution for the following homogeneous linear differential equations:

\[
\begin{align*}
y^{(3)} - 5y'' + 8y' - 4y &= 0 \\
y^{(4)} - y^{(3)} + y'' - 3y' - 6y &= 0 \\
y^{(5)} - y' &= 0 \\
y^{(4)} + 2y'' + y &= 0.
\end{align*}
\]

**Exercise 7:** Find the general solution of the following nonhomogenous differential equations:

\[
\begin{align*}
y'' - y' - 2y &= 3x + 4 \\
y'' + 2y' + 5y &= e^x \sin(x) \\
y''' - 2y'' + y' &= \sin(x) \\
y'' + y &= \sin(x) + x \cos(x) \\
y'' - 4y &= 2e^{2x}
\end{align*}
\]

**Exercise 8:** Use the Wronskian to check whether the following functions are linearly independent on the interval \((-1, 1)\):

\[
\begin{align*}
f(x) &= 1, \quad g(x) = \cos(x), \quad h(x) = \sin(x) \\
f(x) &= x, \quad g(x) = e^x, \quad h(x) = xe^x
\end{align*}
\]