MATH 211: Introduction to Linear Algebra - Fall 2017

Homework 5 – Due date 10/13/17

Read: Section 3.1. Section 1.3 (only Definition 1.3.9 and Example 13).

Important definitions. I recall here some important definitions we have seen in class.

i). A vector $\vec{v}$ in $\mathbf{R}^n$ is a linear combination of the vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_k$ in $\mathbf{R}^n$ if there exist scalars $c_1, c_2, \ldots, c_k$ such that

$$\vec{v} \;=\; c_1\vec{v}_1 + c_2\vec{v}_2 + \ldots + c_k\vec{v}_k.$$

ii). We say that a vector $\vec{v}_i$ in a set of vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_k$ is redundant if $\vec{v}_i$ is a linear combination of the other vectors (namely of $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_{i-1}, \vec{v}_{i+1}, \ldots, \vec{v}_k$). We have seen in class that in order to understand which vectors are non-redundant we need to use the Gauss–Jordan elimination.

Problem 1: i). Write the vector $\begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix}$ as a linear combination of $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$, if possible.

ii). Write the vector $\begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix}$ as linear combination of $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix}$, if possible.

Problem 2: In the following sets of vectors, say which vectors are non-redundant. Moreover write the redundant vectors as linear combination of the others.

$$(i).\quad \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix} \qquad\qquad (ii).\quad \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$(iii).\quad \begin{pmatrix} 1 \\ 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ -1 \\ 1 \\ 1 \end{pmatrix}.$$

Problem 3: Consider the following subset $U = \text{span}\{\begin{pmatrix} 1 \\ 2 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 4 \\ -2 \\ 1 \\ 1 \end{pmatrix}\}$ of $\mathbf{R}^5$.

i). Write U as a span of a minimal set of generators.

ii). Say whether the vector $\begin{pmatrix} 0 \\ 2 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ is in U.

Problem 4: Consider the following vectors $\vec{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $\vec{v}_2 = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$ in $\mathbf{R}^3$.

i). Show that $\vec{v}_1$ and $\vec{v}_2$ are non-redundant vectors.

ii). Say whether the vector $\vec{v} = \begin{pmatrix} 3 \\ 7 \\ 9 \end{pmatrix}$ is in $\text{span}\{\vec{v}_1, \vec{v}_2\}$.

iii). Choose a vector $\vec{v}_3$ in $\mathbf{R}^3$ so that the vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ are non-redundant.

iv). Write $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ as a linear combination of $\vec{v}_1, \vec{v}_2, \vec{v}_3$.

v). Write $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ as a linear combination of $\vec{v}_1, \vec{v}_2, \vec{v}_3$.

Problem 5: i). Write three vectors in $\mathbf{R}^3$ such that their span consists of a line passing through the origin.

ii). Write three vectors in $\mathbf{R}^3$ such that their span consists of a plane passing through the origin.

iii). Write three vectors in $\mathbf{R}^3$ such that their span is equal to $\mathbf{R}^3$ itself.

iv). Is it possible to write 4 distinct non-redundant vectors in $\mathbf{R}^3$? Explain why yes or why not.

Problem 6: Use your own words to answer to the following questions:

- Why is the image of the orthogonal projection in $\mathbf{R}^2$ onto a line L passing thorough the origin equal to L?
- Why is the kernel of the orthogonal projection in $\mathbf{R}^2$ onto a line L passing thorough the origin equal the line perpendicular to L?