Assignment 2: due date February 10th-12th

Problems 1.1: 12, 14: Verify whether the hypotheses of the Theorem on Existence and Uniqueness for initial value problems applies to the following problems. If so, determine a rectangle of the plane as in the statement of the theorem:

\[
\frac{dy}{dx} = x \ln(y); \quad y(1) = 1 \\
\frac{dy}{dx} = 3\sqrt{y}; \quad y(0) = 0
\]

Problem 1.1: 29:
Verify that if \( C \) is a constant, then the functions defined piecewise by
\[
y(x) = \begin{cases} 
0 & \text{for } x \leq C \\
(x - C)^3 & \text{for } x > C
\end{cases}
\]
satisfies the differential equation
\[
\frac{dy}{dx} = 3y^{2/3} \quad \text{for all } x.
\]
Sketch a variety of such solution curves on the \( xy \)-plane. Discuss for which points \((x_0, y_0)\) of the \( xy \)-plane the initial value problem
\[
y(x) = \begin{cases} 
\frac{dy}{dx} = 3y^{2/3} \\
y(x_0) = y_0
\end{cases}
\]
has either a unique solutions, no solutions, infinitely many solutions.

Problem 1.4: 1, 4, 6: Use the technique of separation of variables to solve the following differential equations:

\[
\frac{dy}{dx} + 2xy = 0 \\
(1 + x)\frac{dy}{dx} = 4y \\
\frac{dy}{dx} = 3\sqrt{xy}
\]

Problems 1.4: 23, 25, 27: Find a solution of the following initial value problems:

\[
\frac{dy}{dx} + 1 = 2y; \quad y(1) = 1 \\
x \frac{dy}{dx} - y = 2x^2y; \quad y(1) = 1 \\
\frac{dy}{dx} = 6e^{2x-y}; \quad y(0) = 0
\]

Problems 1.4: 35: Carbon extracted from an ancient skull contained only one-sixth as much \(^{14}\text{C}\) as carbon extracted from present-day bone. How old is the skull?

Problem 1.4: 47: A certain piece of dubious information about phenylethylamine in the drinking water began to spread one day in a city with a population of 100,000. Within a week, 10,000 people had heard this rumor. Assume that the rate of increase of the number who have heard the rumor is proportional to the number who have not yet heard it. How long will it be until half the population of the city has heard the rumor?