WORKSHEET

(1) Find the radius of convergence and interval of convergence of the series (Don't forget to check boundary points)

(a)$\sum_{n=1}^{\infty} \frac{x^n}{n^2}$ **(b)**$\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n^2 5^n}$ **(c)**$\sum_{n=1}^{\infty} \frac{(x-2)^n}{n 4^n}$

(d)$\sum_{n=1}^{\infty} \frac{2^n (x+2)^n}{(n+2)!}$ **(e)**$\sum_{n=1}^{\infty} n!(2x-1)^n$ **(f)**$\sum_{n=1}^{\infty} \frac{(-2)^n (x+3)^n}{\sqrt{n}}$

(2) Find a power series representation for the function and determine the interval of convergence

(a)$f(x) = \frac{1}{1+x^3}$ **(b)**$f(x) = \frac{1}{(1+x)^2}$ **(c)**$f(x) = \frac{x^3}{4x+1}$

(d)$f(x) = ln(5-x)$ **(e)**$f(x) = arctan(\frac{x}{3})$ **(f)**$f(x) = \frac{1}{x^2+25}$

(3)(a) Evaluate the indefinite integral as a power series

$(i)\int \frac{1}{1+x^5}dx$ **(ii)**$\int ln(1+x^4)dx$

(b)Use (a) to approximate the definite integral to three decimal places

$(i)\int_0^{0.2} \frac{1}{1+x^5}dx$ **(ii)** $\int_0^{0.4} ln(1+x^4)dx$

(4) Find the Taylor Series for $f(x)$ centered at the given value of a (Do not show $R_n(x) \to 0$)

(a)$f(x) = 1 + x + x^2 \quad a = 2$ **(b)**$f(x) = e^x$ **a=3** **(c)**$f(x) = sin(x) \quad a = \frac{\pi}{4}$

(d)$f(x) = \sqrt{x} \quad a = 4$ **(e)**$f(x) = xcos(2x) \quad a = 0$ **(f)**$f(x) = xarctan(x) \quad a = 0$

(5)(a)Use the Maclaurin Series for $sin(x)$ to compute $sin(15°)$ correct to three decimal places

(b)Use the Maclaurin Series for e^x to compute $\frac{1}{e}$ correct to three decimal places

(6)(a)Evaluate the indefinite integral as a power series

$(i) \int sin(x^2)dx$ **(ii)**$\int e^{x^3}dx$

(b)Use (a) to approximate the definite integral to three decimal places

$(i) \int_0^1 \sin(x^2)dx$ **(ii)** $\int_0^1 e^{x^3}dx$

(7)Use series to evaluate the limit

(a)$\lim_{x\to 0} \frac{1-cos(x)}{x^2}$ **(b)**$\lim_{x\to 0} \frac{sin(x)-x+\frac{1}{6}x^3}{x^5}$

(8) Find the sum of the series

$(a) \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n}}{n!}$ **(b)**$\sum_{n=0}^{\infty} \frac{3^n}{5^n n!}$

(c)$\sum_{n=0}^{\infty} \frac{n^2}{2^n}$ **(d)**$1 - \ln 2 + \frac{(\ln 2)^2}{2!} - \frac{(\ln 2)^3}{3!}$

(10)Use power series to solve the differential equation

(a)$y' = x^2y$ **(b)**$y'' + x^2y = 0 \qquad y(0) = 1 \qquad y'(0) = 0$