WORKSHEET

(1) Find the radius of convergence and interval of convergence of the series (Don’t forget to check boundary points)

(a) \[ \sum_{n=1}^{\infty} \frac{x^n}{n^2} \]  
(b) \[ \sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n^25^n} \]  
(c) \[ \sum_{n=1}^{\infty} \frac{(x-2)^n}{n4^n} \]  
(d) \[ \sum_{n=1}^{\infty} \frac{2^n(x+2)^n}{(n+2)!} \]  
(e) \[ \sum_{n=1}^{\infty} \frac{n!(2x-1)^n}{\sqrt{n}} \]  

(2) Find a power series representation for the function and determine the interval of convergence

(a) \[ f(x) = \frac{1}{1+x^3} \]  
(b) \[ f(x) = \frac{1}{(1+x)^2} \]  
(c) \[ f(x) = \frac{x^3}{4x+1} \]  
(d) \[ f(x) = \ln(5-x) \]  
(e) \[ f(x) = \arctan(x) \]  
(f) \[ f(x) = \frac{1}{x^3+25} \]  

(3)(a) Evaluate the indefinite integral as a power series

(i) \[ \int \frac{1}{1+x^3} \, dx \]  
(ii) \[ \int \ln(1+x^4) \, dx \]  

(b) Use (a) to approximate the definite integral to three decimal places

(i) \[ \int_0^{0.2} \frac{1}{1+x^3} \, dx \]  
(ii) \[ \int_0^{0.4} \ln(1+x^4) \, dx \]  

(4) Find the Taylor Series for \( f(x) \) centered at the given value of a (Do not show \( R_n(x) \to 0 \))

(a) \( f(x) = 1 + x + x^2 \) \( a = 2 \)  
(b) \( f(x) = e^x \) \( a = 3 \)  
(c) \( f(x) = \sin(x) \) \( a = \frac{\pi}{4} \)  
(d) \( f(x) = \sqrt{x} \) \( a = 4 \)  
(e) \( f(x) = x\cos(2x) \) \( a = 0 \)  
(f) \( f(x) = x\arctan(x) \) \( a = 0 \)

(5)(a) Use the Maclaurin Series for \( \sin(x) \) to compute \( \sin(15^\circ) \) correct to three decimal places

(b) Use the Maclaurin Series for \( e^x \) to compute \( \frac{1}{e} \) correct to three decimal places
(6) (a) Evaluate the indefinite integral as a power series

\[ \int \sin(x^2)\,dx \quad \text{and} \quad \int e^{x^3}\,dx \]

(b) Use (a) to approximate the definite integral to three decimal places

\[ \int_0^1 \sin(x^2)\,dx \quad \text{and} \quad \int_0^1 e^{x^3}\,dx \]

(7) Use series to evaluate the limit

\[ \lim_{x \to 0} \frac{1 - \cos(x)}{x^2} \quad \text{and} \quad \lim_{x \to 0} \frac{\sin(x) - x + \frac{1}{2}x^3}{x^5} \]

(8) Find the sum of the series

\[ \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n}}{n!} \quad \text{and} \quad \sum_{n=0}^{\infty} \frac{3^n}{3^n n!} \]

\[ \sum_{n=0}^{\infty} \frac{n^2}{2^n} \quad \text{and} \quad 1 - \ln 2 + \frac{(\ln 2)^2}{2!} - \frac{(\ln 2)^3}{3!} \]

(10) Use power series to solve the differential equation

\[ y' = x^2y \quad \text{and} \quad y'' + x^2y = 0 \quad y(0) = 1 \quad y'(0) = 0 \]