Practice Midterm 1

MAT 127

Spring 2002

Name:	**ID #:**	**Section:**

Please answer each question in the space provided. Show your work whenever possible. Unless otherwise marked, **answers without justification will get little or no partial credit**. Cross out anything the grader should ignore and circle or box the final answer. *Calculators are not permitted.* You do not have to simplify numerical answers or write their approximate values: if the answer you got is $\sqrt{2}$, you should not replace it by 1.414.

The actual midterm will contain 5 problems, including one word problem. This practice test contains more problems to give you more practice.

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(1) Each of the following three functions solves one of the differential equations below. Which one? Justify your choices.

(a) $y(x) = x \sin x$ (b) $y = e^{2-x^2}$ (c) $y = \frac{x^2+1}{x-1}$

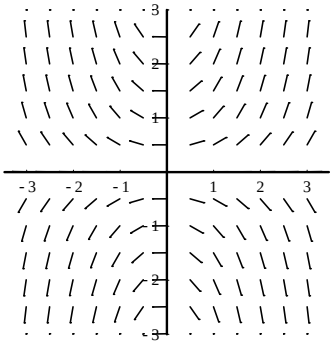
(I) $y' = -2xy$ (II) $xy + y''x = 2y' - 2\sin x$ (III) $1 - y' = \frac{2}{(x-1)^2}$

(2) Match each of the differential equations with their direction field below. (No justification required.)

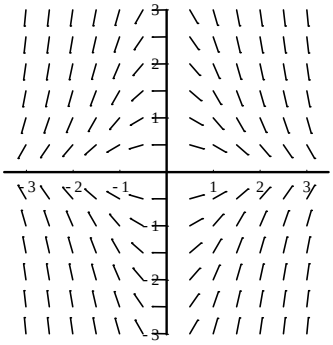
(a) $y' = y(y - 1.5)$ (b) $y' = xy$ (c) $y' = -xy$

(d) $y' = \frac{x}{y}$ (e) $y' = x - y$ (f) $y' = (y - 1)(y + 2)$

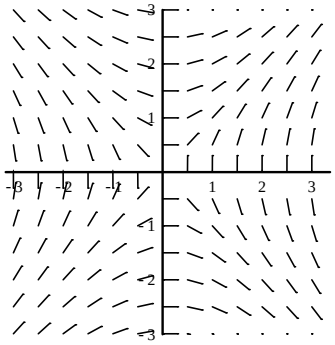
(1)



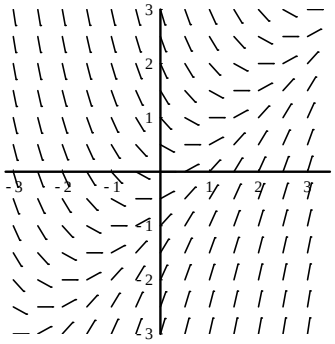
(2)



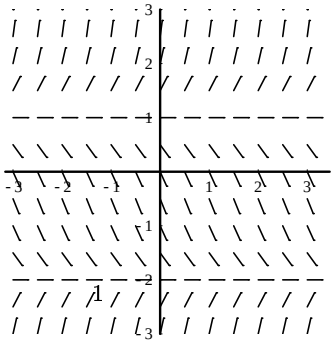
(3)



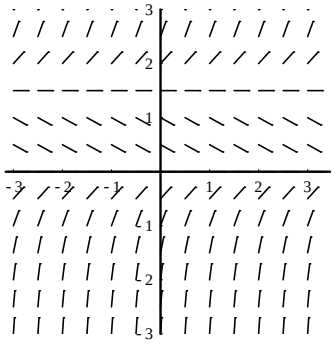
(4)



(5)



(6)



(3) In problem 2 you found the direction field of the differential equation

$$y' = y(y - 1.5).$$

Use this to sketch the three graphs of the solutions to this differential equation that satisfy the initial conditions:

(a) $y(0) = 1$ (b) $y(0) = 2$ (c) $y(-1) = -2.5$

How does the limiting behavior of the solutions depend on the value of $y(0)$? What are the equilibrium solutions? Which one is stable? Which is unstable?

(4) Find the solution of the differential equation

$$y' - 4y = -10\sin(2x)$$

which has the form $y(x) = A\sin(2x) + B\cos(2x)$.

(5) (a) Find all solutions to the differential equation

$$y' = 3y + 15.$$

(b) Solve the initial value problem

$$y' = 3y + 15$$
$$y(0) = -1$$

(6) Find the orthogonal trajectories to the family of curves

$$y = \frac{1}{k + x}.$$

(7) A tank of water initially contains 10 grams of salt dissolved in 10 liters of water. Water is drained from the tank at a rate of 5 liters per hour. Simultaneously, pure water (containing no salt) is added to the tank at a rate of 5 liters per hour. The water in the tank is kept thoroughly mixed, so the salt present is evenly distributed throughout the tank.

(a) Let $y(t)$ be the amount of salt (in grams) in the tank at time t. Write a differential equation for $y(t)$. What are the initial conditions?

(b) Find a solution of the initial value problem of part (a) using the fact (which follows from the general theory we will learn later) that this solution must be of the form $y(t) = Ae^{-rt}$ for some A, r.

(c) How long will it take for the salt concentration to drop to 0.1 gr/liter?