# Practice Final Exam

MAT 127

May 10, 2002

| Name: | ID #: | Section: |
|---|---|---|

**No books, notes, or calculators.**

You do not have to simplify numerical answers or write their approximate values: if the answer you got is $\sqrt{2}$, you should not replace it by 1.414. Double-check your answers, and remember to include units in word problems!

**The real exam will be much shorter;** this is just to give you a lot of practice.

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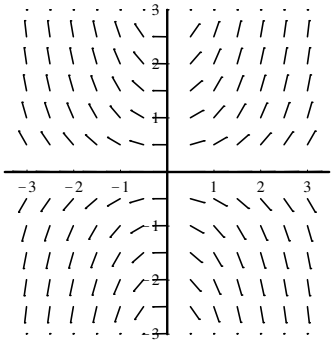
**(1)** Match each of the differential equations with their direction field below. (No justification required.)

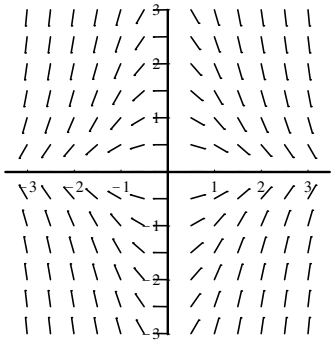
(a) $y' = y(y - 1.5)$ (b) $y' = xy$ (c) $y' = -xy$

(d) $y' = \frac{x}{y}$ (e) $y' = x - y$ (f) $y' = (y - 1)(y + 2)$

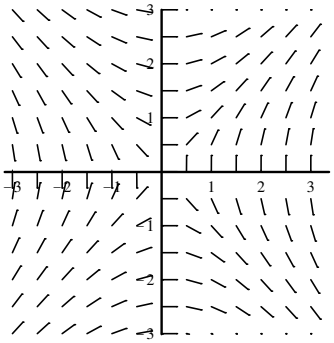
**(1)**



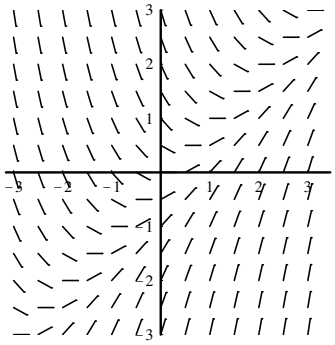
**(2)**



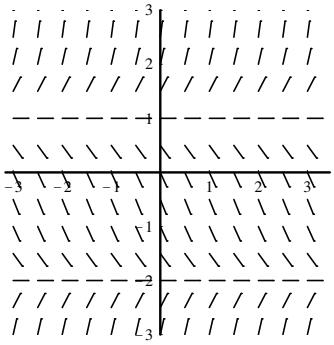
**(3)**



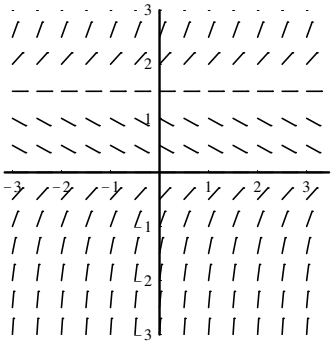
**(4)**



**(5)**



**(6)**



**(2)** Which of the following functions is a solution to the differential equation

$$y'' - y' = 2e^{-x}?$$

(a) $y = e^x + 2e^{-x}$
(b) $y = 2e^x + e^{-x}$
(c) $y = 2 + e^x$
(d) $y = 2e^x + 1$

**(3)** Solve the initial value problem

$$y' = y^2 \cos x, \quad y(0) = 5.$$

**(4)** A turkey is removed from the oven at a temperature of 100°C. The surrounding room is at 20°C. After an hour, the turkey is at 60°C. Assuming that the turkey obeys Newton's law of cooling, find the time at which the turkey temperature is 30°C.

**(5)** A tank contains 1000L of brine with 15kg of dissolved salt. Pure water enters the tank at a rate of 10L/min. The solution is kept thoroughly mixed and drains from the tank at the same rate. How much salt is in the tank after 20 minutes?

**(6)** Determine whether each of the following series is convergent or divergent. Justify each answer and state which test (Integral, Ratio, p-series, Comparison, etc.) you are using.
NOTE: If the series is convergent, you **do not** need to find its sum.

(a) $\sum_{n=1}^{\infty} \cos(n\pi/2)$

(b) $\sum_{n=1}^{\infty} \frac{1}{n!}$

(c) $\sum_{n=3}^{\infty} \frac{n}{2n^2+3}$

(d) $\displaystyle\sum_{n=1}^{\infty} \cos(e^{-n})$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n3^n}$

(f) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\ln(n^2)}{n}$

**(7)** For each of the following power series, find the interval of convergence. *Note that this could be a single point, all of* $\mathbb{R}$*, or an interval of the form* $(a, b)$*,* $[a, b]$*,* $[a, b)$*, or* $(a, b]$*.*

(a) $\displaystyle\sum_{n=0}^{\infty}(x-5)^n$

(b) $\displaystyle\sum_{n=0}^{\infty}3^n x^n$

(c) $\displaystyle\sum_{n=1}^{\infty}n!x^n$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(x+2)^n}{n \ln n}$

**(8)** Compute the Taylor series for each of the following functions $f(x)$ about the given point $a$.

(a) $f(x) = e^x, \quad a = 2$

(b) $f(x) = \dfrac{1}{1+x}, \quad a = 0$

(c) $f(x) = \sin(x^3), \quad a = 0$

(d) $f(x) = \dfrac{1}{(1-x)^3}, \quad a = 0$

(e) $f(x) = (x-1)^3 \ln x, \quad a = 1$

**(9)** Find the sum of the series.

(a) $1 - \ln 3 + \dfrac{(\ln 3)^2}{2!} + ... = \displaystyle\sum_{n=0}^{\infty} \frac{(-\ln 3)^n}{n!}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{5^n n!}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{(2n)!}$

**(10)** Compute the indefinite integral $\int \frac{\sin x}{x}$ as an infinite series, then evaluate $\int_0^1 \frac{\sin x}{x}$ correct to within 3 decimal points.

**(11)** Use a Taylor series to compute $1/\sqrt{e}$ to within an error of $1/100$ (give the answer as a fraction). Is your approximation an overestimate or an underestimate?

**(12)** Use Maclaurin series for $\sin x$ and $\cos x$ to approximate the following numbers with ratio of integers,

(a) $\sin 1$ up to the 3rd decimal point

(b) $\cos \dfrac{1}{2}$ correct to within $\frac{1}{100}$

**(13)** (a) Find a power series solution to the initial value problem $y' = xy, y(0) = 1$.
(b) Solve the equation explicitly (note that it's separable) to find the solution of the initial value problem.
(c) Identify the 2 solutions from (a) and (b), to obtain a power series expansion of the function you found in (b).

**(14)** Use series to evalute the limits

(a) $\lim_{x \to 0} \dfrac{\sin x}{x}$

(b) $\lim_{x \to 0} \dfrac{1 - \cos x}{e^x - 1 - x}$