Remarkable lives an legacy of Sofia Kovalevskaya and Emmy Noether

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Plan

1. Sofia Vasilyevna Kovalevskaya
   - Early years
   - Higher mathematics
   - Major work

Plan

Sofia Vasilyevna Kovalevskaya

Sofia Kovalevskaya, 1850–1891

Sofia Kovalevskaya (née **Korvin-Krukovskaya**), was born in Moscow on 15 January 1850. Her father, lieutenant general **Vasily Vasilyevich Korvin-Krukovsky**, was a head of the Moscow artillery and her mother, **Yelizaveta Fedorovna Schubert**, was from a family of German scholars who had settled in Saint Petersburg during the time of Catherine the Great.

Sofia's maternal grandfather was general **Theodor Friedrich von Schubert**, a head of the Russian military topographic service.

Theodor Friedrich von Schubert, 1789–1865

Sofia had a typical upbringing for a girl of her class and time. She was left largely in the care of nurses and governesses, spoke English and French almost as well as she did Russian, and was reared in the belief that her future would be settled by the marriage with a young man of suitable wealth and family position.

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- Gauss-Ostrogradsky theorem

$$\iiint_V (\nabla \cdot \boldsymbol{F}) dV = \oiint_S (\boldsymbol{F} \cdot \boldsymbol{n}) dS$$

Childhood memories

"As I speak of these, my first contacts with mathematics, I cannot help mentioning a curious circumstance which also helped to arouse my interest in the field. When we moved permanently to the country, the whole house had to be redecorated and all the rooms had to be freshly wallpapered. But since there were many rooms, there wasn't enough wallpaper for one of the nursery rooms... But by happy chance, the paper for this preparatory covering consisted of the lithographed lectures of Professor Ostrogradsky on differential and integral calculus, which my father had acquired as young man. These sheets, all speckled over with strange, unintelligible formulas, soon attracted my attention; I remember as a child standing for hours on end in front of this mysterious wall, trying to figure out at least some isolated sentences and to find the sequence in which the sheets should follow one another. From this protracted daily contemplation, the outer appearance of many of these formulas imprinted themselves in my memory; indeed, their very text left a deep trace in my brain, although they were incomprehensible to me while I was reading them."

First mathematics lessons

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- In order to study abroad, Sofia needed written permission from her father (or husband). Accordingly, she contracted a 'fictitious marriage' with Vladimir Kovalevsky in 1868.

Vladimir Onufrievich Kovalevsky, 1842–1883

- Vladimir Kovalevsky was a Russian revolutioner (belonged to the Russian nihilist movement', a kind of utopian socialism), geologist, paleontologist, founder of evolution paleonthology.

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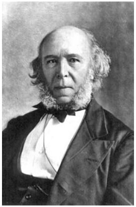

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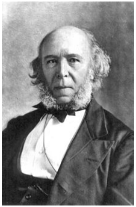

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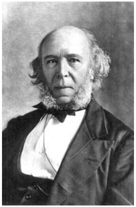

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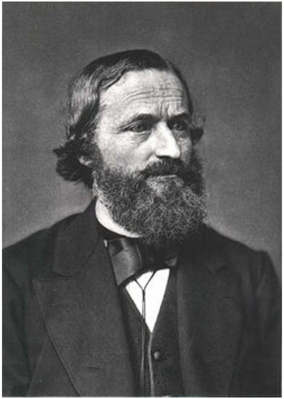
- 1871 — took private lessons with Karl Weierstrass in Berlin (the university did not allow to audit classes).

Hermann Ludwig Ferdinand von Helmholtz, 1821–1894



Gustav Robert Kirchhoff, 1824–1887

Karl Theodor Wilhelm Weierstrass, 1815–1897

- Called Sofia the most talented of his students (who included such eminent mathematicians as Georg Frobenius, Hermann Schwarz and Carl Runge) and watched out for her interests as carefully as he did for his own.

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- Paper on local existence and uniqueness theorem for Cauchy initial value problem for partial differential equations with analytic coefficients — the celebrated Cauchy-Kovalevskaya theorem.
- Unknown to Weierstrass and Kovalevskaya, Cauchy proved a special case of this theorem in 1842. Starting with Poincaré, Hermite and others it was acknowledged that Kovalevskaya's elegant method proves the general case. She also noted that certain equations have no solutions even when they have "formal power series"" solutions, famous Kovalevskaya example.

Theorem (Cauchy-Kovalevskaya)

Consider the initial value problem

$$\partial_t^m u = G(t,x,\partial_t^j \partial_x^\alpha u), \quad 0 \le j \le m-1;\ j+|\alpha| \le m$$
$$\partial_t^i u(0,x) = g_j(x), \quad 0 \le j \le m-1.$$

Suppose that g_j *are real analytic on a neighborhood of* $x_0 \in \mathbb{R}^d$, *and* G *is real analytic on a neighborhood of*

$$(0, x_0, \partial_t^j \partial_x^\alpha g_j(x_0), \quad 0 \le j \le m-1; j+|\alpha| \le m).$$

Then there is a real analytic solution defined on a neighborhood of $(0,x_0) \in \mathbb{R} \times \mathbb{R}^d$. *The solution is unique in the sense that if* u *and* v *are both real analytic solutions to the equation on a connected neighborhood of* $(0,x_0)$, *then* $u = v$.

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- The most immediate barrier to her mathematical career, however, was the social convention of the time.
- Kovalevskaya was married, and married women did not live apart from their husbands, nor did they support themselves with teaching positions.
- Even Weierstrass thought Kovalevskaya was doing mathematics for the intellectual satisfaction. Her husband would support her and she does not need official recognition of her mathematical accomplishments.

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- In order to teach at the higher levels, one needed a Russian master's degree. But women were forbidden to take the exam to obtain the degree.
- She plunged into literary circles, tried her hand at writing, was active in the movement to establish a women's university in St. Petersburg, and so on.

Back to mathematics

Gösta Mittag-Leffler, 1846–1927

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- * — following Weierstrass.

Paris, December 18, 1888

Permanent secretaries of the Academy to Madam Sofya Kovalevskaya in Stockholm

Madam,

We have the honour to notify You that the Academy of Sciences awarded You the Prix Bordin (improvement in an important point of the theory of motion of a solid body). We invite You, Madam, to be present at a public session that will be held on Monday, December 24 of this year at one o'clock p.m. exactly and at which time the results of the competition will be announced in public. We take this opportunity to convey our personal congratulations and testify to our confidence of the benefit the Academy anticipates Your work and Your advances will bring. Madam, receive our assurances in our highest respect.

L. Pasteur J. Bertrand

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- Died in Stockholm in 1891 of pneumonia.

Kovalevskaya case

- Differential equations

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\begin{aligned}
2\dot{p} &= qr, & \dot{\gamma} &= r\gamma' - q\gamma'', \\
2\dot{q} &= -pr - c\gamma'', & \dot{\gamma}' &= p\gamma'' - r\gamma, \\
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Kovalevskaya case

- Differential equations

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- Solutions are meromorphic functions of complexified time $t \in \mathbb{C}$.

Legacy

- Along with Weierstrass, Hermite, Mittag-Leffler, Picard and Poincaré, Sofia Kovalevskaya was considered one of the best mathematical analysts in Europe.

Pelageya Kochina "*Love and Mathematics: Sofia Kovalevskaya*"
Mir Publishers, Moscow, 1985

Amalie Emmy Noether

Amalie Emmy Noether, 1882–1935

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Max Noether, 1844–1921

- Her mother, **Ida Amalia Kaufmann**, the daughter of a wealthy Jewish merchant family.

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- Hilbert responded with indignation, stating, "I do not see that the sex of the candidate is an argument against her admission as privat dozent. After all, we are a university, not a bath house."

David Hilbert, 1862–1943

Noether theorem

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- In classical physics Noether theorem uses equations of motion while its analog in quantum physics deals with expectation values of quantum fields.

Abstract algebra

- **Noetherian ring** — a ring that satisfies the ascending chain condition on ideals: for any such chain of ideals

  $$I_1 \subseteq I_2 \cdots \subseteq I_{k-1} \subseteq I_k \subseteq I_{k+1} \subseteq \cdots$$

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- In 1935 Emmy Noether died and was buried in the walkway surrounding the cloisters of Bryn Mawr's Library.

Tribute to Emmy Noether

$$S=\int \mathscr{L}(\varphi,\partial_\mu\varphi)d^4x.$$

Principle of the least action: $\delta S=0$ yields Euler-Lagrange equations

$$\frac{\partial\mathscr{L}}{\partial\varphi}-\frac{\partial}{\partial x^\mu}\frac{\partial\mathscr{L}}{\partial(\partial_\mu\varphi)}=0.$$

Energy-momentum tensor:

$$\theta^\mu_\nu=\frac{\partial\mathscr{L}}{\partial(\partial_\mu\varphi)}\partial_\nu\varphi-\delta^\mu_\nu\mathscr{L}.$$

Noether theorem

If the action S is invariant under the infinitesimal transformations $\Delta x^\mu=X^\mu_\nu\varepsilon^\nu$ and $\Delta\varphi=\Phi_\mu\varepsilon^\mu$, then the Noether current

$$J^\mu_\nu=\frac{\partial\mathscr{L}}{\partial(\partial_\mu\varphi)}\Phi_\nu-\theta^\mu_\lambda X^\lambda_\nu$$

is conserved:

$$\partial_\mu J^\mu_\nu=0.$$