MAT 127 Calculus C Fall 2002
Practice Midterm II

Name:____

I.D.:________________________ **Section number:** ________

Please answer each question in the space provided and on the reverse side of the sheets. Show your work whenever possible. Unless otherwise indicated, **answers without justification will get little or no partial credit!** Cross out anything that grader should ignore and circle or box the final answer. You **do not need** to simplify numerical answers or write their approximate values: if the answer you got is $\sqrt{2}$ you should not replace it by 1.414. Pay close attention to the distinction between sequences of numbers and series (which are sums). The actual exam will contain 5 problems. This practice test contains more problems to give you more practice.

1. Determine whether each of the following sequences converges.

(a)

$$\left\{\frac{\sqrt{n}}{\sqrt{n}+1}\right\}_{n=1}^{\infty}$$

(b)

$$\left\{\frac{n^2+2}{n+1000}\right\}_{n=5}^{\infty}$$

(c)

$$\left\{e^{1+1/n}\right\}_{n=1}^{\infty}$$

(d)

$$\left\{\frac{\sin^2 n}{n}\right\}_{n=1}^{\infty}$$

2. Determine whether each of the following series is convergent or divergent. Justify your answer and state which test (Integral, Comparison, p-Series, etc.) you are using. NOTE: if the series converges, you do not need to find its sum.

(a)

$$\sum_{n=1}^{\infty}\frac{10000}{1+2^n}$$

(b)

$$\sum_{n=1}^{\infty}\frac{3^{n+5}}{4^n}$$

(c)

$$\sum_{n=0}^{\infty}(-1)^n n$$

(d)

$$\sum_{n=10}^{\infty}\frac{5}{n^2-2n+1}$$

(e)

$$\sum_{n=1}(-1)^n n^{-3/2}$$

(f)

$$\sum_{n=1}^{\infty}\frac{\cos(2n^2)}{n^2}$$

(g)

$$\sum_{n=5}^{\infty}\frac{1}{n\ln n}$$

3. Determine whether the following series is convergent or divergent.

$$\sum_{n=1} n^2 e^{-n^3}$$

4. It is given that

$$\frac{\pi^2}{6}=\sum_{n=1}^{\infty}\frac{1}{n^2}.$$

Let $A = 1+1/2^2+1/3^2+\cdots+1/1000^2$. Estimate the difference $|\pi^2/6 - A|$. Is A greater than or less than $\pi^2/6$?

5. Express the number $5.404040404040\ldots$ as a ratio of integers.

6. A sequence is defined recursively as

$$a_1 = 1,\ \ a_{n+1} = a_n/2n.$$

Determine whether the series

$$\sum_{n=1}^{\infty} a_n$$

converges.

7. A hard rubber ball has the property that after being dropped from a height h onto a hard surface, it bounces back up to a height rh, where $r < 1$. If the ball is dropped from an initial height of H meters, and continues to bounce indefinitely, what will be the total distance that the ball travels?