

# MAT 127 Calculus C Spring 2003 Practice Midterm II

Name: \_\_\_\_\_

I.D.: \_\_\_\_\_ Section number: \_\_\_\_\_

Please answer each question in the space provided and on the reverse side of the sheets. Show your work whenever possible. Unless otherwise indicated, **answers without justification will get little or no partial credit!** Cross out anything that grader should ignore and circle or box the final answer. You **do not need** to simplify numerical answers or write their approximate values: if the answer you got is  $\sqrt{2}$  you should not replace it by 1.414. Pay close attention to the distinction between sequences of numbers and series (which are sums). The actual exam will contain 5 problems. This practice test contains more problems to give you more practice.

1. Determine whether each of the following sequences converges.

(a)

$$\left\{ \frac{\sqrt{n}}{\sqrt{n} + 1} \right\}_{n=1}^{\infty}$$

(b)

$$\left\{ \frac{n^2 + 2}{n + 1000} \right\}_{n=5}^{\infty}$$

(c)

$$\left\{ e^{1+1/n} \right\}_{n=1}^{\infty}$$

(d)

$$\left\{ \frac{\sin^2 n}{n} \right\}_{n=1}^{\infty}$$

(e)

$$\left\{ \frac{\ln(n^2)}{n} \right\}_{n=1}^{\infty}$$

2. Determine whether each of the following series is convergent or divergent. Justify your answer and state which test (Integral, Comparison,  $p$ -Series, etc.) you are using. NOTE: if the series converges, you do not need to find its sum.

(a)

$$\sum_{n=1}^{\infty} \frac{10000}{1 + 2^n}$$

(b)

$$\sum_{n=1}^{\infty} \frac{3^{n+5}}{4^n}$$

(c)

$$\sum_{n=0}^{\infty} (-1)^n n$$

(d)

$$\sum_{n=10}^{\infty} \frac{5}{n^2 - 2n + 1}$$

(e)

$$\sum_{n=1}^{\infty} (-1)^n n^{-3/2}$$

(f)

$$\sum_{n=1}^{\infty} \frac{\cos(2n^2)}{n^2}$$

(g)

$$\sum_{n=5}^{\infty} \frac{1}{n \ln n}$$

4. Find the sum of the series

(a)

$$\sum_{n=1}^{\infty} \frac{3^n + 2^n}{6^n}$$

(b)

$$\sum_{n=1}^{\infty} \frac{2}{n^2 + 4n + 3}$$

4. Determine whether the following series is convergent or divergent.

$$\sum_{n=1}^{\infty} n^2 e^{-n^3}$$

5. It is given that

$$\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2}.$$

Let  $A = 1 + 1/2^2 + 1/3^2 + \cdots + 1/1000^2$ . Estimate the difference  $|\pi^2/6 - A|$ . Is  $A$  greater than or less than  $\pi^2/6$ ?

6. Show that the series is convergent. How many terms of the series do we need to add in order to find the sum with indicated accuracy?

(a)

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \quad (|\text{error}| < 0.0001)$$

(b)

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}} \quad (|\text{error}| < 0.000005)$$

7. Express the number  $5.\overline{40} = 5.4040404040\dots$  as a ratio of integers.
8. A sequence is defined recursively as

$$a_1 = 1, \quad a_{n+1} = \frac{n+1}{2n+3} a_n$$

Determine whether the series

$$\sum_{n=1}^{\infty} a_n$$

converges.

8. A hard rubber ball has the property that after being dropped from a height  $h$  onto a hard surface, it bounces back up to a height  $rh$ , where  $r < 1$ . If the ball is dropped from an initial height of  $H$  meters, and continues to bounce indefinitely, what will be the total distance that the ball travels?