MAT 542 Assignment No. 7 Due: April 21, 2003

1 Show that

(a)

$$\frac{1}{z^2} = 1 + \sum_{n=1}^{\infty}(n+1)(z+1)^n$$

for $|z+1| < 1$.

(b)

$$\frac{1}{z^2} = \frac{1}{4} + \frac{1}{4}\sum_{n=1}^{\infty}(-1)^n(n+1)\left(\frac{z-2}{2}\right)^n$$

for $|z-2| < 2$.

2 Prove the representation

$$\frac{z}{(z-1)(z-3)} = -\frac{1}{2(z-1)} - 3\sum_{n=1}^{\infty}\frac{(z-1)^n}{2^{n+1}}$$

and determine the region in which it is valid.

3 Show that $f(z) = \sum_{n=0}^{\infty} z^{2^n}$ is the only power series with $f(0) = 0$ satisfying $f(z) = z + f(z^2)$.

4 Evaluate $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}$. (*Hint:* Use Abel's theorem).

5 (a) Does there exist a meromorphic function on $\mathbb{C}$ satisfying $f(n) = \infty$ for $n = 1, 2, \ldots$?

(b) Does there exist an entire function f satisfying $f(\frac{1}{n}) = \frac{n}{n-1}$ for $n = 1, 2, \ldots$? (*Hint:* Use the uniqueness theorem for holomorphic functions).

(c) Does there exist an entire function f such that $f^{(n)}(0) = 2^n n!$ for $n = 0, 1, 2\ldots$?

(d) Does there exist an entire function f such that $f^{(n)}(0) = (n!)^2$ for $n = 0, 1, 2\ldots$?

6 Problem 5 on p. 184 in Ahlfors.

7 Problem 2 on p. 186 in Ahlfors (only the sketch of the proof is required).

8 Problem 4 on p. 186 in Ahlfors.

9 Problem 5 on p. 186 in Ahlfors.