MAT 542 Assignment No. 5 Due: March 24, 2003

(1) Evaluate the integral

$$\int_\gamma \frac{\cos z}{(z-\pi)^3}dz$$

in the following cases:

a) $\gamma = \{z(\theta) = 4e^{i\theta}\ ,\ 0 \le \theta \le 2\pi\}$

b) $\gamma = \{z(\theta) = 3e^{i\theta}\ ,\ 0 \le \theta \le 2\pi\}$

(2) Let f be holomorphic at each point of $\mathbb{C}$ and suppose that there is an integer n and two positive real numbers R and M such that $|f(z)| \le M|z|^n$ for $|z| \ge R$. Show that $f(z)$ is a polynomial of degree at most n.

(3) Problem No. 5 on p. 123 in Ahlfors.

(4) Problem No. 6 on p. 123 in Ahlfors.

(5) Let $f(z)$ be holomorphic in $0 < |z| < 1$. If $|f(z)| \le M|z|^\alpha$ for $0 < |z| < 1$ (for some $M > 0$, and some $\alpha \in \mathbb{R}$) what can you conclude about the singularity of f at $z = 0$?

(6) Let f be an entire function (holomorphic function on $\mathbb{C}$). In each case, prove or give a counter-example.

a) $\lim_{z\to\infty} f(z) = 0 \Longrightarrow f$ is constant.

b) If there is a sequence z_n such that

$$\lim_{n\to\infty} z_n = \infty \quad \text{and} \quad \lim_{n\to\infty} f(z_n) = 0$$

then f is constant.

(7) Problem No. 4 on p. 130 in Ahlfors.

(8) Problem No. 5 on p. 130 in Ahlfors.

(9) Let g be holomorphic in $0 < |z| < 1$. Prove or give counter-examples.

a) If g has a removable singularity at $z = 0$, so does e^g.

b) If g has a pole at $z = 0$, so does e^g.

c) If g has an essential singularity at $z = 0$, so does e^g.

(10) Problem 2 on p. 136 in Ahlfors.

(11) Problem 3 on p. 136 in Ahlfors.

(12) Problem 4 on p. 136 in Ahlfors.

(13) Problem 5 on p. 136 in Ahlfors. In addition, explicitly describe the group of all one-to-one conformal mappings of the unit disk $|z| < 1$ and the upper half-plane $\Im z > 0$.