MAT 535: HOMEWORK 11

Due THU May 5

Problems marked by asterisk (*) are optional and will not be graded. Problems marked by ($\star$) are for extra credit.

1. Let R be a subring of the commutative ring S with 1. Prove that the integral closure of R in S is integrally closed in S.

***2.** Let $p > 2$ be a prime and let ζ_p be a primitive p-th root of 1. Prove that $1, \zeta_p, \ldots, \zeta_p^{p-1}$ is a basis of the ring $\mathcal{O}_K$ of algebraic integers in the cyclotomic field $K = \mathbb{Q}(\zeta_p)$.

3. D&F, Exercises 3, 6*, 8, and 17 on pp. 852–853.

4. Exercises 2*, 5, 10 and 23 on pp. 876–879.

$\star$ **5.** Let R be a subring of the polynomial ring $\mathbb{C}[x]$ which contains at least one non-constant element. Prove that the integral closure of R in $\mathbb{C}[x]$ is $\mathbb{C}[x]$.