MAT 534: HOMEWORK 5
DUE TU, Oct. 6

Problems marked by asterisk (*) are optional.

1. Let G be abelian group, which is generated by x, y, z with relations $x^3 = xy^2z^3 = 1$. Represent G as a product of cyclic groups.

  Hint: You can write G additively and work out Smith normal form.

2. Same question for the group $\mathbb{Z}^3/L$, where L is generated by $u_1 = (3, 2, 5), u_2 = (0, 1, 3), u_3 = (0, 1, 5)$.

3. Let A be $n \times n$ matrix with integer coefficients, and let $L \leq \mathbb{Z}^n$ be the subgroup generated by the rows of A. Prove that $\mathbb{Z}^n/L$ is infinite if $\det A = 0$ and if $\det A \neq 0$, $|\mathbb{Z}^n/L| = |\det A|$. (You can use existence of Smith normal form).

*4. From Dummit and Foote, problems 2 and 3 on p. 165.

5. From Dummit and Foote, problem 8 on p. 166.

*6. From Dummit and Foote, problem 14 on p. 167.