MAT 534: HOMEWORK 4
DUE THU SEP 24

1. Let $K, L \leq G$ be such that L normalizes K, i.e., $lKl^{-1} = K$ for all $l \in L$. Prove that $KL \leq G$ and $KL \cong K \rtimes L/K \cap L$.
2. Describe all Sylow 2-subgroups and 3-subgroups of D_{12}.
3. Prove that if $|G| = 105$, then G has a normal Sylow 5-subgroup and a normal Sylow 7-subgroup. Prove that if G has a normal Sylow 3-subgroup, then G is abelian.
4. Let G be a group of order p^2q, where p, q are primes, $p < q$. Assume that p does not divide $q - 1$. Prove that then G is abelian.
5. Classify all groups of order 75.
6. Classify all groups of order 20.