EXTRA PROBLEMS FOR HOMEWORK 2

1. Determine whether the Fourier series of the following functions converge uniformly or not. Sketch each function.
   (a) $f(x) = e^x$, $-1 < x < 1$;
   (b) $f(x) = x + |x|$, $-\pi < x < \pi$;
   (c) $f(x) = 1 + 2x - 2x^3$, $-1 < x < 1$.

2. The Fourier series of the function

$$f(x) = \frac{\sin x}{x}, \quad -\pi < x < \pi,$$

converges at every point. To what value does the series converge at $x = 0$? at $x = \pi$? The convergence is uniform. Why?

3. Let a_n and b_n be Fourier coefficients of the function $f(x)$, $-\pi < x < \pi$. If a_n and b_n tend to zero as n tends to infinity, show that the series

$$a_0 + \sum_{n=1}^{\infty} e^{-\alpha n}(a_n \cos nx + b_n \sin nx),$$

where $\alpha > 0$, converges uniformly.