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Ludwig Dmitrievich Faddeev (obituary)

In February 2017 Ludwig Dmitrievich Faddeev, one of the greatest modern mathematicians and theoretical physicists, passed away. Faddeev's death is a heavy loss to Russian and international science. His work has largely reshaped modern mathematical physics, to the development of which he devoted his life. Many of his results have become classical, belonging to the gold reserve of pure mathematics, and at the same time playing a decisive role in the development of the most important areas of theoretical physics. He also made enormous contributions to filling the decades-long gap between mathematics and physics and forming several generations of Russian and international theorists. For more than 60 years, Faddeev's scientific life was associated with the Leningrad (later St. Petersburg) Branch of the Steklov Mathematical Institute (LOMI for short, and later POMI), where he went from a postgraduate student to the head of a laboratory he himself organized, and then to the director of the institute. Faddeev was as proud of the creation of the Laboratory of Mathematical Problems in Physics, where he brought together students who had grown under his guidance, as of his purely scientific results. In 1976,

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at the age of 42, he was elected a full member of the Academy of Sciences of the USSR, and from 1992 until the end of his life he was the head of the Mathematics Department (later the Division of Mathematical Sciences) of the Russian Academy of Sciences.

Faddeev's life was both happy and dramatic. He was able to realize many of the dreams of his youth. However, at various times the fundamental science to which he devoted his life appeared to be regarded as unnecessary to the authorities, who kept science on 'hunger rations' and then demonstratively imposed a humiliating 'reform' of the Academy of Sciences, against the wishes of the scientific community. Faddeev spoke with great sorrow of these events. He maintained his opposition to incompetent authorities for many years in various forms. In the 1990s, it was expressed in his struggle for preserving the Euler International Mathematical Institute he had organized. The stressful situations concerning this institute contributed to Faddeev suffering a heart attack.

Ludwig Faddeev was born in Leningrad on March 23, 1934, to the family of the remarkable mathematicians Dmitrii Konstantinovich Faddeev and Vera Nikolaevna Faddeeva. Dmitrii Faddeev was one of the leading Soviet algebraists and the founder of the Leningrad school of modern algebra. For many years Vera Faddeeva headed the LOMI Laboratory of Computational Methods, which she herself created. Dmitrii was also a brilliant pianist, and both he and Vera hoped for a musical career for their son. Although these plans could not be realized because of World War II, Ludwig always maintained a profound interest in music. On entering university, he decided to enroll in the Faculty of Physics of Leningrad State University instead of the Faculty of Mathematics, where his father was a dean at that time. He mentioned afterwards that this decision reflected his non-conformism and his desire to make his own way in life and science. Olga Aleksandrovna Ladyzhenskaya was his main teacher in the Department of Mathematics of the Faculty of Physics. Apart from delivering several mandatory courses, she had organized a student seminar, where Faddeev was one of the main speakers. He prepared his Ph.D. thesis (a complete solution of the inverse scattering problem for the one-dimensional Schrödinger operator) under the supervision of Ladyzhenskaya at LOMI, an institution closely associated with his whole subsequent scientific life. He always remained deeply grateful to her and kept her portrait on the writing desk in his study at POMI until the last day of his life.

From his student years Faddeev was attracted by the mathematical beauty and intricacy of quantum field theory. However, in the mid-1950s, after the great successes connected with the construction of quantum electrodynamics, quantum field theory encountered serious difficulties and 'went out of fashion' for a long period of time. People tried to overcome these difficulties, on the one hand, by recovering the scattering amplitude from its postulated analytic properties and, on the other hand, by developing an abstract axiomatic approach. Neither of these approaches attracted Faddeev, who held that mathematical (and, in particular, geometric) beauty should be a crucial feature of correct models free from the difficulties that undermined trust in the very existence of quantum field theory. Doing quantum field theory at that time meant working alone, against the mainstream. To establish a basis, Faddeev wanted first of all to solve a technically difficult problem.

This was the quantum-mechanical three-body problem, where Faddeev made decisive progress in the early 1960s. This work (which became the subject of his D.Sc. thesis) was of great interest to theorists dealing with applied aspects of quantum physics (the theory of atomic collisions, nuclear theory) and gave rise to a truly immense literature. At the same time it was a striking technical achievement from the point of view of pure mathematics. This was one of the first cases when a differential operator with complicated structure of the continuous spectrum was exhaustively investigated. Interestingly, one of the main ideas of that paper (a transformation of the integral equations of scattering theory into the so-called Faddeev equations, making it possible to control the contribution of the continuous spectrum) occurred to Faddeev in his study of the Thirring model, the simplest (but already highly non-trivial) model in quantum field theory. In the next decade, this same Thirring model again played an important role in the development of the quantum inverse scattering method. The work on multiparticle scattering was continued by Faddeev's students and colleagues (V. S. Buslaev, O. A. Yakubovskii, and S. P. Merkur'ev), but Faddeev himself, after completing his now-famous monograph on the quantum three-body problem, moved on resolutely to another, even more difficult subject: quantum field theory.

The experience of working with differential operators whose continuous spectrum has a complicated structure turned out to be useful in another problem studied by Faddeev in the mid 1960s at the initiative of I. M. Gelfand: the theory of automorphic functions (for discrete groups with a non-compact fundamental domain of finite area on the Lobachevskii plane). Faddeev was the first to give a complete non-arithmetic proof of the eigenfunction expansion theorem and (jointly with A. B. Venkov and V. L. Kalinin) the Selberg trace formula for non-compact Riemann surfaces of finite area. Together with B. S. Pavlov, he proposed an interesting non-stationary formulation of the scattering problem for automorphic functions which enables one to interpret zeros of the Riemann zeta function as quantum-mechanical resonances.

In 1966 Faddeev and V. N. Popov made crucial progress in the construction of quantum Yang–Mills theory. Their paper was fully appreciated several years later, when the Yang–Mills theory became the basis for the theory of electroweak and strong interactions in elementary particle theory. The method of Feynman path integrals used by Faddeev and Popov became a working tool of the new theory. Faddeev liked to say that he was able to outdo Feynman at his own game.

Faddeev and Popov noted already in the first paper that their method is rather general. To give another application, they stated rules for quantizing gravitational fields. A method for quantizing gravitation was outlined there and then developed in a series of papers. (Remarkably, rearrangement of path integrals to a Hamiltonian form gave them a simple way to obtain a Hamiltonian formulation of gravitation theory. This is important not only in quantum theory, but also in classical theory, including the problem of positive energy. In the 1960s Faddeev obtained one of the best results of that time on the positivity problem. This problem was completely solved 10 year later by S.-T. Yau and R. Schoen, and then more simply by E. Witten.) Unfortunately, the successes of perturbation theory in quantum gravitation were very limited because of its non-renormalizability. However, as soon as the ideas of quantum field theory were adopted in other areas of mathematics (representation theory and topology), it became increasingly clear that the Faddeev–Popov method is universal and flexible.

The inverse scattering method was an important theme of Faddeev's research since the late 1950s. At that time he generalized the Gelfand–Levitan–Marchenko results to the case of the Schrödinger operator on the whole axis. He also studied the spectral trace identities (together with Buslaev) and made significant advances in the three-dimensional inverse problem. Faddeev regarded the solution of the three-dimensional inverse problem as his best analytic result. In the late 1960s and early 1970s, the inverse scattering theory on the line unexpectedly appeared to be connected with the exciting successes in the study of non-linear equations. From the very beginning, Faddeev was one of the main actors of this 'heroic period' when the volume of our knowledge about integrable systems used to double between two conferences. His very first paper on this theme immediately became classical. Together with V. E. Zakharov he showed that the Korteweg–de Vries equation is an infinite-dimensional Liouville-integrable Hamiltonian system. Before their paper no non-trivial examples of infinite-dimensional integrable systems (that is, systems of field type) had been known. The importance of the Zakharov–Faddeev paper was enormous: it showed that infinite-dimensional integrable systems exist and are very interesting and non-trivial. Of special interest was an explicit description of the phase space of the Korteweg–de Vries equation in terms of the 'scattering data'. It distinguished the contribution of the famous soliton solutions (solitary waves) behaving like true particles. As Zakharov later said, this was a paradigm shift.

All the topics that occupied Faddeev's attention in the previous period happily came together in the theory of integrable non-linear equations: the inverse scattering problem, trace identities, and Hamiltonian mechanics. At the same time, his contribution to the development of the inverse scattering method was not limited to his purely technical achievements. His role of a teacher and organizer was no less important. This was the period when he headed a group of young coworkers that had formed in his laboratory (mainly graduates from the Faculty of Physics and the Faculty of Mathematics and Mechanics at Leningrad University who had attended his seminar at LOMI). It was a great honour for anyone to belong to this group.

From the very beginning, Faddeev's interest in non-linear equations had to do more with their possible applications to quantum field theory than with their role as useful models in the mechanics of continuous media. In this respect, a very important role was played by the sine-Gordon equation, whose Lax pair and 'action-angle' variables Faddeev and L. A. Takhtajan constructed in 1973. This result gave rise to a surprising (but still quasi-classical) picture for the spectrum of masses in a relativistic problem of quantum field theory: besides the particles corresponding to the principal field, there are also solitons and their bound states. The solitons carry a non-trivial charge and may naturally be regarded as strongly interacting. The sine-Gordon equation was the first to indicate the role of particle-like classical solutions and, more generally, the richness of correctly chosen non-linear models in quantum field theory. This paper had a considerable ideological influence on theoretical physics in the 1970s and 80s. The soliton solutions in multidimensional theories ('t Hooft–Polyakov monopoles) and instanton solutions in the Yang–Mills theory grew from it.

Along with the quasi-classical quantization of solitons, another theme of the 1970s was a search for mathematical tools giving exact solutions of two-dimensional quantum models. This search gave rise to a new splendid synthesis: a construction (jointly with E. K. Sklyanin and Takhtajan) of the quantum inverse scattering method, which combined the ideas of the classical inverse scattering method with the methods and results of R. Baxter and H. Bethe. Surprisingly, the exact solution of the quantum-field Hamiltonian model involved the 'bootstrap' scattering amplitudes (the S-matrix of A. B. and Al. B. Zamolodchikov) that were studied in the 1960s as an alternative to the Hamiltonian methods in field theory. A byproduct of this new development was the theory of quantum groups, which stemmed from the study of the quantum Yang–Baxter equation (first recognized as an interesting algebraic object in papers by Faddeev and his students) and, after works of V. G. Drinfeld, became one of the most interesting topics of non-commutative algebra since the discovery of Lie groups. Jointly with Takhtajan and N. Yu. Reshetikhin, Faddeev developed a convenient general approach to quantization of Lie groups and algebras which was completely based on the use of quantum R-matrices. This approach led to a number of important advances: the construction of quantum algebras corresponding to lattices and arbitrary graphs, as well as the quantum deformation of the Knizhnik–Zamolodchikov equations.

A quasi-classical construction related to the method of R-matrices (also referred to as the theory of classical r-matrices) was first proposed by Sklyanin and enabled one to algebrize the classical inverse scattering method. At the same time, it appeared to be very interesting from the point of view of symplectic and Poisson geometry. In all these results, the role of Faddeev as an actively working mathematician and mentor was pivotal.

Another thing on the agenda in the late 1980s and early 1990s was a synthesis between the Hamiltonian approach in the quantum inverse scattering method and conformal field theory. Conformal theories are naturally covariant under finite-dimensional quantum groups, and this was realized as a part of the (infinite-dimensional) hidden symmetry of integrable models. Faddeev's papers of that period contributed to a clarification of the hidden quantum symmetry and led to new interesting lattice models of field theory (in discrete space-times), which may also be viewed as non-trivial examples of 'non-commutative geometry' in the spirit of A. Connes.

In the 1980s, the work on the quantum inverse scattering method somewhat shunted aside the 'four-dimensional' physics in Faddeev's laboratory, but a number of important results (some of which are still 'reserved' for the future) were obtained in this direction as well. First and foremost, this concerns papers by Faddeev and S. L. Shatashvili on quantum-field anomalies and the discovery of an interesting class of group cohomology and the related extension of the three-dimensional group/algebra of gauge transformations. Another direction which Faddeev pursued at that time was the search for non-trivial solutions of four-dimensional non-linear equations, where the role of the topological charge was played by the Hopf invariant. This research was continued in the 1990s in collaboration with the Finnish physicist A. Niemi, who confirmed by means of numerical methods the existence of stable 'knotted' solutions of the equations of the modified Skyrme model proposed in Faddeev's papers.

The break-up of the USSR and the deep crisis of the country brought about profound changes in the life of Russian science. As a result, many coworkers in the laboratory organized by Faddeev left for leading American and European universities. Some of his closest collaborators (A. G. Izergin, A. P. Oskolkov, and Popov) passed away untimely due to the stressful situations in the 1990s. In those years he himself often worked abroad, but his fundamental decision was to stay in Russia. In particular, he declined an offer to become the head of the Institute of Theoretical Physics at Stony Brook after the retirement of its long-term director, the Nobel prize laureate C.-N. Yang. His constant concern was the survival of fundamental science and the fates of the Mathematical Institute and the Division of Mathematical Sciences of the Russian Academy of Sciences.

The new conditions also gave rise to some interesting new possibilities of collaboration. We have already mentioned joint papers by Faddeev and Niemi. The Lagrangian of the model with knotted solitons can be obtained from the fundamental Lagrangian of Yang–Mills fields by certain changes of variables. It is hoped that the solitons with non-trivial Hopf invariant correspond to the hypothetical collective excitations of Yang–Mills fields in quantum chromodynamics. Up to the last months of his life, Faddeev was considering yet another theme related to Yang–Mills fields: the problem of emergence of masses. He was always somewhat skeptical about the now-standard way of introducing masses into the Yang–Mills theory by means of the so-called 'Higgs mechanism'. He wanted to connect it with a correct renormalization procedure producing a dimensional constant that determines mass scaling in a theory which originally contained only dimensionless parameters. Faddeev repeatedly returned to this theme. His last paper (written jointly with S. E. Derkachev and A. V. Ivanov, Faddeev's last graduate student) is also devoted to a description of the renormalization scenario for Yang–Mills fields.

The quantum inverse scattering method remained one of the most important themes in the 1990s and 2000s. It has found unexpected applications in high-energy physics. When the transmitted momentum is large, the relativistic scattering amplitudes become effectively two-dimensional (this has been well known since the 1960s) and can be studied using methods developed in the theory of two-dimensional integrable models. This was discussed in papers by L. N. Lipatov, Faddeev, and G. P. Korchemskii. Elaboration of more profound aspects of the quantum inverse scattering problem gave rise to the study of infinite-dimensional realizations of quantum R-matrices and to the very important concept of modular duality for quantum Lie groups and algebras. Faddeev's paper on modular doubles of quantum groups is a deep 'reserve' for future investigations relating to non-trivial new chapters in representation theory, non-commutative geometry, the theory of special functions (q-dilogarithms), quantum gravitation theory, and many other topics whose study is now only beginning.

Another non-trivial application of the quantum inverse scattering method involves Yang–Mills theories. At the dawn of the theory of integrable systems there was a romantic hope that the Yang–Mills theory could also be integrable. This is false, but a version of it (the so-called supersymmetric Yang–Mills theory) is indeed close to integrability or exact solubility. N. A. Nekrasov and Shatashvili recently discovered that describing the vacuum sector in the supersymmetric Yang–Mills theory (in particular, in four-dimensional space-time) immediately leads to quantum integrable systems. All the basic ingredients of the quantum inverse scattering method find their places in this new approach to supersymmetric gauge theories, and active work in this area is still in progress. This astonishing connection between seemingly unrelated aspects of Faddeev's scientific heritage is perhaps one of the best evidences of its depth.

Faddeev's awards included a Heinemann Prize in mathematical physics (1974), a Dirac Medal (1990), a Max Planck Medal (1996), a Pomeranchuk Prize (2002), a Poincaré Prize (2006), a Shaw Prize (2008, jointly with V. I. Arnold), a USSR State Prize (1971), State Prizes of the Russian Federation (1995, jointly with A. A. Slavnov, and 2005), a Demidov Prize (2002), an Euler Medal (2002), and a Lomonosov Medal of the Russian Academy of Sciences (2013). The establishment of an international medal in honour of Ludwig Faddeev was announced at the 23d International Conference on Many-Particle Quantum Theory in August 2016. He was elected a foreign member of a number of academies and scientific societies, including the Paris Academy of Sciences (since 2002), the London Royal Society (since 2010), and the National Academy of Sciences of the USA (since 1990).

The heritage of Ludwig Dmitrievich Faddeev and his ideas remain with us and will determine the development of mathematical and theoretical physics for many decades to come.

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