SKETCH OF SOLUTIONS (HOMEWORK VII)

1.- No: $(123)(12)(123)^{-1} = (23) \notin H$

8.- a $\{(12)(13)\}[(12)(34)]\{(12)(13)\}^{-1} = (13)(24) \notin H$

b Coset multiplication is not well defined

11.- Follows immediately from associativity.

25.- Yes, H and K are isomorphic: $1 \mapsto 1$, $15 \mapsto 9$ is an isomorphism. $G/K \cong \mathbb{Z}_2 \times \mathbb{Z}_2 \ncong \mathbb{Z}_4 \cong G/H$

37.- $(aH)(bH) = abH = baH = (bH)(aH)$

Chapter 10

5.- $x^{rs} = (x^r)^s$, odd r yield an isomorphism (we need the homomorphismo to be injective and surjective)

7.- $\sigma\phi(g_1 g_2) = \sigma(\phi(g_1)\phi(g_2)) = \sigma\phi(g_1)\sigma\phi(g_2)$

19.- Since Φ is not injective KerΦ is a nontrivial subgroup of $\mathbb{Z}_{17}$ therefore Ker$\Phi = Z_1 7$ i.e. $\Phi \equiv e$

20.- There is no homomorphism onto $(8 \nmid 20)$. Any homomorphism ϕ is determined by $\phi(1)$ since 1 generates $\mathbb{Z}_{20}$ by the orders of the groups the only possible homomorphisms are determined by:

$$1 \mapsto 0$$
$$1 \mapsto 2$$
$$1 \mapsto 4$$
$$1 \mapsto 6$$

29.- {7,17}

35.- $(xy)^6 = x^6 y^6$. Ker$\Phi = e^{2\pi ik/6}$ for $k = 0, 1, 2, 3, 4, 5$