SKETCH OF SOLUTIONS (HOMEWORK VI)

1.- H, $\alpha_5 H$, $\alpha_9 H$ Notice that there must be 3 cosets, and different cosets are disjoint.

7.- $|U(30)| = 8$ and $|H| = 2$ threfore there must be 4 cosets, by inspection we get H, $7H$, $13H$ and $19H$

15.- Let H be a proper subgroup of G then $|H| < |G| = pq$ and by Lagrange's theorem $|H| \mid pq$, so $|H|$ equals either 1, p or q and by corollary 3 this implies H is cyclic.

3.-

$$5^{2\cdot 7+1} \equiv (5^7)^2 \cdot 5 \equiv 5^2 \cdot 5 \equiv 4 \cdot 5 \equiv 6 \mod 7$$

$$7^{11+2} \equiv 7 \cdot 7^2 \equiv 7 \cdot 5 \equiv 2 \mod 11$$

19.- $g^m = e$ therefore $|g| \mid m$ but since $(m,n) = 1$ If $|g| \mid n$, then $|g| = 1$. Using Lagrange's theorem we get that $|g| = 1$ therefore $g = e$

25.- $1, 3, 11, 33$. The number of elements of order 11 must be a multiple of 10 *why?* Therefore since $33 \equiv 3 \mod 10$ there must be at least two elements g and h of order not equal to 11 or 1. If the order of one of these elements (say g) is 33 then $|g^{11}| = 3$ and we are done.

31.-

$$\text{stab}(1) = \{(1), (24)(56)\} \;\; \text{orb}(1) = \{1, 2, 3, 4\}$$
$$\text{stab}(3) = \{(1), (24)(56)\} \;\; \text{orb}(3) = \{1, 2, 3, 4\}$$
$$\text{stab}(1) = \{(1), (12)(34), (13)(24), (14)(23)\} \;\; \text{orb}(5) = \{5, 6\}$$

42.- Label the points in the diagrams in the following way:

$$\begin{matrix} a & b & c \\ d & e & f \end{matrix}$$

$$\text{stab}(a) = \{R_0, H\} \;\; \text{stab}(d) = \{R_0\}$$
$$\text{stab}(b) = \{R_0, D'\} \;\; \text{stab}(e) = \{R_0\}$$
$$\text{stab}(c) = \{R_0, H\} \;\; \text{stab}(f) = \{R_0\}$$

45.- 50. By Lagrange's theorem $10 \mid |G|$, $25 \mid |G$, therefore $\text{lcm}(10, 25) \mid |G$ therefore $50 \mid |G|$. Since $|G| < 100$ we get $50 = |G|$