SKETCH OF SOLUTIONS (HOMEWORK V)

Chapter 5

3. Write each permutation as a product of disjoint cycles and apply Ruffini's theorem: a) 3, b) 3, c) 6, d) 12, e) 12, f) 2.
5. By Ruffini's theorem we only need to compute $\text{lcm}(4, 6) = 12$
7. Any element of S_n can be expressed as a product of λ disjoint cycles of lengths $m_1, \ldots, m_\lambda$. Therefore all possible orders k of elements of S_n are of the form
$$k = \text{lcm}(m_1, \ldots, m_\lambda)$$
with $1 \le m_i \le n$ and $m_1 + \ldots m_\lambda \le n$ Analogously, any element of A_n has order k if
$$k = \text{lcm}(m_1, \ldots, m_\lambda)$$
with $1 \le m_i \le n$ and $m_1 + \ldots m_\lambda \le n$ and $\sum_{i=1}^{\lambda} m_i$ is odd. By inspection we get that the orders of elements of S_n are $1, 2, 3, 4, 5, 6$; for A_6 the orders are $1, 2, 3, 4, 5$; for S_7 the orders are $1, 2, 3, 4, 5, 6, 7, 10, 12$ and for A_7 the orders are $1, 2, 3, 4, 5, 6, 7$.
11. If n es odd then an n-cycle is an even permutation. If n is even then an n-cycle is an odd permutation.
15. Even.
31. (1) $\text{Id} \in \text{stab}(a)$: Clear since $\text{Id}(a) = a$
    (2) Suppose $\alpha, \beta \in \text{stab}(a)$ then $\alpha\beta^{-1} \in \text{stab}(a)$: Since $\alpha(a) = a$ and $\beta(a) = a$, $\beta^{-1}(a) = \beta^{-1}\beta(a) = a$ and $\alpha\beta^{-1}(a) = \alpha(a) = a$.

Chapter 6

1. Define $\phi : \mathbb{Z} \to 2\mathbb{Z}$ by $\phi(a) = 2a$. $\phi(a) = 2a$. ϕ is an isomorphism with inverse $2a \mapsto a$
5. Define $\phi : \text{U}(8) \to \text{U}(12)$ by $1 \mapsto 1, 3 \mapsto 5, 5 \mapsto 7, 7 \mapsto 11$
22. They are not isomorphic. In U(20) $|3| = 4$, but every element in U(24) has order 2. (The order of an element is preserved under isomorphisms).
31. (1) One to one: Suppose $\log(a) = \log(b)$ then $10^{\log(a)} = 10^{\log(b)}$ i.e. $a = b$
    (2) Onto: Let $x \in \mathbb{R}$ then $10^x = e^{x \ln(10)} > 0$ and $\log(10^x) = x$
    (3) $\log(ab) = \log(a) + \log(b)$