# MAT 126 Calculus B Spring 2005
# Practice Final Exam

Answer each question in the space provided and on the reverse side of the sheets. Show your work whenever possible. Unless otherwise indicated, **answers without justification will get little or no partial credit!** Cross out anything that grader should ignore and circle or box the final answer. You **do not need** to simplify numerical answers or write their approximate values. This practice exam contains more problems then the actual test to give you more practice.

**1.** Evaluate the following definite integrals:

(a)
$$\int_1^9 \ln \sqrt{x}\, dx$$

(b)
$$\int_0^2 \frac{x}{1+2x^2}\, dx$$

(c)
$$\int_1^e \frac{(\ln x)^3}{x}\, dx$$

(d)
$$\int_{-1}^1 x^2 \sin(x^5)\, dx$$

(e)
$$\int_0^{1/2} \frac{\sin^{-1} x}{\sqrt{1-x^2}}\, dx$$

(f)
$$\int_1^4 \sqrt{t} \ln t\, dt$$

(g)
$$\int_0^{13} \frac{dx}{\sqrt[3]{(1+2x)^2}}$$

(h)
$$\int_0^{\frac{\pi}{2}} \sin^3 x\, dx$$

**2.** Evaluate the following indefinite integrals:

(a)
$$\int x^2 e^x\, dx$$

(b)
$$\int \frac{2x^3+1}{x^2+1}\,dx$$

(c)
$$\int \frac{\tan^{-1} x}{1+x^2}\,dx$$

(d)
$$\int \sin^{-1} x\,dx$$

(e)
$$\int \frac{x-1}{x^2+3x+2}\,dx$$

(f)
$$\int t^2\cos(1-t^3)\,dt$$

(g)
$$\int e^x\sqrt[3]{1+e^x}\,dx$$

(h)
$$\int \cos^5 x\,dx$$

**3.** (a) Write a formula for $\cos^2 x$ in terms of $\sin^2 x$.
(b) Evaluate
$$\int \cos^3 x \sin^2 x\,dx$$

**4.** Let
$$f(x) = \int_2^{\sqrt{x}} \frac{\sin t}{t}dt + x^2$$
(a) Find $f'(x)$.
(b) Evaluate $f(4)$.

**5.** Find a function $f$ and a number $a$ such that for $x$,
$$1 + \int_a^x tf(t)\,dt = x^3$$

**6.** (a) Let
$$I = \int_0^4 e^{x^2}dx$$
For any value of $n$ list the numbers $L_n, R_n, M_n, T_n$ and $I$ in increasing order.

(b) Repeat part (a) for

$$I = \int_0^{\sqrt{2}/2} e^{-x^2} dx$$

**7.** Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

(a)

$$\int_0^{\infty} e^{-x}\, dx$$

(b)

$$\int_0^1 \frac{1}{\sqrt{x}}\, dx$$

(c)

$$\int_0^3 \frac{1}{x\sqrt{x}}\, dx$$

(d)

$$\int_{-\infty}^{\infty} xe^{-x^2}\, dx$$

(e)

$$\int_0^1 \frac{1}{4y-1}\, dy$$

**8.** Find the area of the region bounded by the curves:

(a) $y = x^2$ and $y = x^4$.

(b) $x + y^2 = 2$ and $x + y = 0$.

**9.** (a) Find the volume of the solid of revolution obtained by rotating the region bounded by the curves $y = x^2$ and $y^2 = x$ about the $x$-axis.

(b) Find the volume of the solid of revolution obtained by rotating the region bounded by $y = \sec x$, $y = 1$, $x = -1$ and $x = 1$ about the $x$-axis.

**10.** Find the length of the following curves:

(a) $y = x^{3/2}$, $0 \le x \le 2$.

(b)

$$y = \frac{x^2}{4} - \frac{\ln x}{2}, \quad 1 \le x \le 2$$

**11.** Find the average value $f_{ave}$ of $f$ on the given interval.

(a) $f(x) = x\sin(x^2)$ on $[0, \sqrt{\pi}]$.

(b) $f(x) = 4 - x^2$ on $[0, 3]$.

(c) For $f$ as in part (b) find the number $c$ in $[0, 3]$ such that $f(c) = f_{ave}$.