Answer each question in the space provided and on the back of the sheets. Write full solutions, not just answers: unless otherwise marked, answers without justification will get little or no partial credit. Cross out anything the grader should ignore and circle or box the final answer. Do NOT round answers.

No books, notes, or calculators!

1. Compute the following limits by distinguishing between “lim $f(x) = \infty$”, “lim $f(x) = -\infty$”, and “limit does not exist even allowing for infinite values”.

(a) $\lim_{x \to -1} x^3 + 7x^2 - 1$
(b) $\lim_{x \to \infty} x \tan \frac{1}{x}$
(c) $\lim_{x \to 0} \frac{x^2 - 4x - 5}{x - 5}$
(d) $\lim_{x \to 0} x^4 \cos \frac{\pi}{x}$
(e) $\lim_{x \to 0} \frac{e^x - 1 - x - x^2/2}{x^3}$
(f) $\lim_{x \to \infty} \frac{x^3 + 1001x + 77}{x^3 - x^2 + 99}$
(g) $\lim_{x \to \pi/2} \frac{\cos x}{2x - \pi}$
(h) $\lim_{x \to \infty} (xe^{1/x} - x)$

2. Compute the derivatives of the following functions

(a) $f(x) = x^3 - 12x^2 + x + 137\pi$
(b) $f(x) = (2x + 1) \sin x$
(c) $g(s) = \sqrt{1 + e^{2s}}$
(d) $h(t) = \frac{1 + e^t}{1 - e^t}$
(e) $f(x) = (2x + 2)^{100}$
(f) $g(x) = x^{\sin x}$

3. Let $f(x) = xe^{-x^2}$.

(a) Find asymptotes of $f(x)$ (hint: $f(x) = \frac{x}{e^{x^2}}$)
(b) Compute the derivative of \( f(x) \)

(c) On which intervals is \( f(x) \) increasing? decreasing?

(d) Sketch a graph of \( f(x) \) using the results of the previous parts and the fact that \( f(0) = 0 \).

4. Let \( f(x) = -2x^3 + 6x^2 - 3 \).

   (a) Compute \( f' \), \( f'' \).

   (b) On which intervals is \( f(x) \) increasing/decreasing?

   (c) On which intervals is \( f(x) \) concave up/down?

   (d) Find all critical points of \( f(x) \). Which of them are local maximums? local minimums? neither? Justify your answer.

5. It is known that the polynomial \( f(x) = x^3 - x - 1 \) has a unique real root. Between which two whole numbers does this root lie? Justify your answer.

6. It is known that for a rectangular beam of fixed length, its strength is proportional to \( w \cdot h^2 \), where \( w \) is the width and \( h \) is the height of the beam’s cross-section. Find the dimensions of the strongest beam that can be cut from a 12’’ diameter log (thus, the cross-section must be a rectangle with diagonal 12’’).

7. The curve defined by the equation

\[
y^2(y^2 - 4) = x^2(x^2 - 5)
\]

is known as the “devil’s curve”. Use implicit differentiation to find the equation of the tangent line to the curve at the point \( (0; -2) \).

8. Find the most general function \( f(x) \) satisfying

   (a) \( f''(x) = \cos x \)

   (b) \( f'(x) = \frac{x^2 + x + 1}{x} \)