Section 7.3

2. $\dfrac{dy}{dx} = \dfrac{e^{2x}}{4y^2}$

$$4y^3 dy = e^{2x} dx$$

$$\int 4y^3 dy = \int e^{2x} dx$$

$$y^4 = \frac{1}{2} e^{2x} + C$$

---

8. $\dfrac{dz}{dt} + e^{t+z} = 0$

$$\frac{dz}{dt} = -e^{t+z}$$

$$= -e^t e^z$$

$$e^{-z} dz = -e^t dt$$

$$\int e^{-z} dz = \int -e^t dt, so\ that$$

$$-e^{-z} = -e^t + C \qquad or\ \text{explicitly},$$

$$\mathrm{z} = -\ln|\mathrm{e}^{\mathrm{t}} - C|$$

---

10. $\dfrac{dy}{dx} = \dfrac{y\cos x}{1+y^2}, \quad y(0) = 1$

$$\frac{1+y^2}{y} dy = \cos x dx$$

$$\int \frac{1+y^2}{y} dy = \int \cos x dx$$

$$\left(\int \frac{1+y^2}{y} dy = \int \frac{1}{y} + y\ dy \right)$$

$$\ln|y| + \frac{1}{2} y^2 = \sin x + C$$

$Let \quad x=0 \ \ and \ \ y=1.$

$Then \ \ \ln 1+\frac{1}{2}\cdot 1^2 = \sin 0 + C$

$So \ that \qquad C=\frac{1}{2}$

Therefore, $\ln|y|+\frac{1}{2}y^2 = \sin x + \frac{1}{2}$

---

16. (1,1) is a given point, and $y'=\frac{dy}{dx}$ is the slope at (x, y)

Thus

$$\frac{dy}{dx}=\frac{y^2}{x^3}$$

$$Then \quad \frac{dy}{y^2}=\frac{dx}{x^3}$$

$$\int\frac{dy}{y^2}=\int\frac{dx}{x^3},$$

$$-\frac{1}{y}=-\frac{1}{2x^2}+C$$

Put the point $(1,1)$ to this equation.

$$-\frac{1}{1}=-\frac{1}{2\cdot 1}+C, \quad then \ C=-\frac{1}{2}$$

$$Hence, \quad -\frac{1}{y}=-\frac{1}{2x^2}-\frac{1}{2}=-\frac{1+x^2}{2x^2}$$

$$Therefore, \quad y=\frac{2x^2}{1+x^2}$$

38.

$$Note: \frac{d^2s}{dt^2}=\frac{dv}{dt}, \ that \ is, \ \frac{d}{dt}\left(\frac{ds}{dt}\right)=\frac{d}{dt}v, \quad So \ that, \ \frac{ds}{dt}=v$$

a) $\quad m\frac{dv}{dt}=-kv$

$$\frac{dv}{v} = -\frac{k}{m} dt$$

$$\int \frac{dv}{v} = \int -\frac{k}{m} dt$$

$$\ln|v| = -\frac{k}{m} t + C$$

$$v = e^{-\frac{k}{m}t+C} = e^{-\frac{k}{m}t} e^{C}$$

Put the point (0, v_0) to that equation.

$$v_0 = e^C, \ so \quad C = \ln|v_0|$$

$$Hence, \qquad v = v_0 e^{-\frac{k}{m}t}$$

$$\frac{ds}{dt} = v_0 e^{-\frac{k}{m}t}$$

$$s = \int v_0 e^{-\frac{k}{m}t} dt = -\frac{mv_0}{k} e^{-\frac{k}{m}t} + C_2$$

Put the point (0, s_0) to that equation.

$$s_0 = -\frac{mv_0}{k} + C_2, \quad so \quad C_2 = s_0 + \frac{mv_0}{k}$$

$$Hence, \qquad s = -\frac{mv_0}{k}\left(e^{-\frac{k}{m}t} - 1\right) + s_0$$

Moreovre, the travel distance after timet is

$$s(t)\text{-}s(0) = -\frac{mv_0}{k} e^{-\frac{k}{m}t}$$

b) $m\frac{dv}{dt} = -kv^2$

$$\frac{dv}{v^2} = -\frac{k}{m}dt$$

$$\int \frac{dv}{v^2} = \int -\frac{k}{m}dt$$

$$-\frac{1}{v} = -\frac{k}{m}t + C$$

Put the point (0, v_0) to that equation.

$$-\frac{1}{v_0} = -\frac{k}{m} \cdot 0 + C = C$$

Hence,

$$-\frac{1}{v} = -\frac{k}{m}t - \frac{1}{v_0} = -\frac{kv_0t + m}{mv_0}$$

$$so \quad v = \frac{mv_0}{kv_0t + m}$$

$$\frac{ds}{dt} = \frac{mv_0}{kv_0t + m}$$

$$s = \int \frac{mv_0}{kv_0t + m}dt$$

$$= \frac{m}{k}\ln|kv_0t + m| + C_2$$

Put the point (0, s_0) to that equation.

$$s_0 = \frac{m}{k}\ln|m| + C_2, \quad so \quad C_2 = -\frac{m}{k}\ln|m| + s_0$$

Hence,

$$s = \frac{m}{k}\ln|kv_0t + m| - \frac{m}{k}\ln|m| + s_0$$

$$= \frac{m}{k}\ln\left|\frac{kv_0t + m}{m}\right| + s_0$$

Moreover, the travel distance after time t is

$$s(t) - s(0) = \frac{m}{k}\ln\left|\frac{kv_0t + m}{m}\right|$$