Section 7.1

2.

$$y = \sin x \cos x - \cos x$$

$$y(0) = \sin 0 \cos 0 - \cos 0$$

$$= -1$$

and

$$y' = (\sin x \cos x)' - (\cos x)'$$

$$= \cos^2 x - \sin^2 x + \sin x$$

Substitute y' as a function of x to the given equation.

Then the given equation is written as following.

$$(\cos^2 x - \sin^2 x + \sin x) + (\tan x)(\sin x \cos x - \cos x)$$

$$= \cos^2 x - \sin^2 x + \sin x + \frac{\sin x}{\cos x}(\sin x \cos x - \cos x)$$

$$= \cos^2 x$$

Then y satisfies the given equation

4.

$y = e^{rt}$ *then*

$y' = re^{rt}$

$y'' = r^2 e^{rt}$

Then

$$r^2 e^{rt} + re^{rt} - 6e^{rt} = 0$$

$$(r^2 + r - 6)e^{rt} = 0$$

However, $e^{rt} > 0$ for all real number r

So that

$$(r+3)(r-2) = 0$$

Therefore $r = 2, -3$

12.

There exists a solution in the give A, B, C because of the hypothesis of the problem.
Thus it suffices to eliminate the answers which are not possible.
In the given graph, $y'(0) > 0$
($y'(0)$ is the slope of the tangent line of the graph at the point $(0,\ y(0))$.)

$A.\ \ y'(0) = 1 + 0y(0) = 1 > 0$
$B.\ \ y'(0) = \ -2y'(0) = 0$
$C.\ \ y'(0) = 1 - 0y'(0) = 1 > 1$

So B is not possible answer.

In the first quadrant, there is a point of the given graph whose slope of the tangent line is 0. In other words, for some point $(a,\ y(a))$

$$y'(a) = 0\ ,\ \ where\ \ a,\ y(a) > 0$$

$A.\ \ y' = 1 + xy$
If x and y are positive then y'>0, that is, for all point $(a,\ y(a))$ in the first quadrant, $y'(a) > 0$

Hence, A cannot be a possible answer.

Therefore, the (possible) answer is C.