

PRACTICE MIDTERM II, FOR MAT 127

(1) Let $y(t)$ denote the weight of a certain radioactive substance at time t (where 1 mg=unit of weight, and 1 year= unit of time). If the “half life” of this substance is 20 years, and there is 10mg of the substance at time $t=0$, then at what time will there be only 1mg of the substance left?

(2) The differential equation

$$P' = 3P(1 - .01P)$$

describes the growth of a population of bears within a fenced in forest.

- (a) Find the equilibrium solutions to this differential equation.
- (b) Explain in common sense terms what the equilibrium solutions tell us about the growth of the bear population.
- (c) Sketch the direction field for this differential equation, and include in this sketch the equilibrium solutions.

(3) Indicate which of the following sequences converge and which diverge. Give an explanation (or show computations) for each answer.

- (a) a_1, a_2, a_3, \dots where $a_n = \ln(n^2)/n$.
- (b) b_1, b_2, b_3, \dots where $b_n = 2 + \cos(n\pi)$.
- (c) c_1, c_2, c_3, \dots where $c_n = (n \cos(n))/(n^2 + 1)$.

(4) A sequence a_1, a_2, a_3, \dots is defined by setting $a_1 = 1$ and requiring that $a_n = 4 + a_{n-1}/3$ hold for all $n > 1$.

- (a) Show that $\{a_n\}$ is an increasing sequence.
- (b) Find an upper bound for this sequence.
- (c) Does this sequence converge or diverge (explain)? If it converges what does it converge to?

(5) Determine which of the following series converge and which diverge . (Give an explanation or show your computations.)

- (a) $3+3/2+3/4+3/6+3/8+3/10+\dots$
- (b) $3-3/2+3/4-3/6+3/8-3/10+\dots$
- (c) $\sum_{n=1}^{\infty} 10(2/3)^n$
- (d) $\sum_{n=1}^{\infty} (-1)^n (\sqrt[2]{n})/(\sqrt[2]{n} + 1)$

(6)

- (a) State the “Integral Test” for a series.
- (b) State the “Comparison Test” for a two series.

(7) Use the Integral Test and the Comparison Test to determine which of the following series converge.

- (a) $\sum_{n=1}^{n=\infty} 1/(n\sqrt{n} + 2)$
- (b) $\sum_{n=1}^{n=\infty} (n + 1)/n^3$
- (c) $\sum_{i=1}^{i=\infty} 1/(i\ln(i))$