

Sec. 1.5 6 The derivative of a periodic function is periodic.

Proof. Let f be a periodic function with period T , i.e. $f(x + T) = f(x)$ for any x in the domain. Then

$$\frac{d}{dx}f(x) \Big|_{x=x_0} = \frac{d}{dx}f(x + T) \Big|_{x=x_0} = \frac{d}{dx}f(x) \Big|_{x=x_0+T}$$

□

The integral of a periodic function is usually not periodic. For example, 1 is periodic but $\int 1 = x$ is not.

Sec 1.6 2 First of all,

$$x \sim \sum_{n=1}^{\infty} \frac{\sin(n\pi x)}{n}$$

It is also easy to see

$$\int_{-1}^1 x = \frac{2}{3}$$

By fomula 5 on page 87,

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

Then it is easy to check that Parseval's equality.

For $\sin x$ it is extremely easy since $\sin x$ is its own Fourier series.