Homework 7: §4.1: 1-4 §4.2: 1-4, 7, 8, 10

See back of book for computations from §4.1.

Exercises 4.2

1-3. See back of book.

4. Multiply the right side of the equality $(\pi\sigma)^2 = \pi^2\sigma^2$ by σ^{-1} to obtain $\pi\sigma\pi = \pi^2\sigma$. Now multiply the left side by π^{-1} to get $\sigma\pi = \pi\sigma$

7. To do this, we express each as a product of disjoint cycles. (1 2 3)(2 3 4) = (1 2)(3 4), which is order $lcm(2, 2) = 2$. (1 2 3)(3 2 4) = (1 2 4), which is order 3. (1 2 3)(3 4 5) = (1 2 3 4 5), which is of order 5.

8. It remains to prove parts (ii) and (iii).

(ii) $(\pi^r)^s = \pi^{rs}$

Proof: Induction on s. The case $s = 1$ is trivial. Suppose the claim holds for $s = k - 1$. From (i) we can write $(\pi^r)^k = (\pi^r)^{k-1}(\pi^r)$. By the inductive hypothesis, we then have $(\pi^r)^k = \pi^{r(k-1)}\pi^r$. But now we can again apply (i), and so $\pi^{r(k-1)}\pi^r = \pi^{r(k-1)+r} = \pi^{rk}$, establishing the claim in the case $s = k$. By the principle of induction, the claim holds for all s.

(iii) $\pi^{-r} = (\pi^r)^{-1}$

Proof: Induction on r. The case $r = 1$ is trivial. Suppose the claim holds for $r = k-1$. Then by definition (using (i)) $\pi^{-k} = (\pi^{-1})^k = (\pi^{-1})^{k-1}(\pi^{-1}) = \pi^{-(k-1)}\pi^{-1}$. Now we can use the inductive hypothesis to conclude $\pi^{-k} = (\pi^{k-1})^{-1}\pi^-1$. Using the hint we see $= (\pi\pi^{k-1})^-1 = (\pi^k)^{-1}$, as desired.

10. It's enough to show that we can constuct any transposition. We can then proceed as in 4.2.10 and 4.2.11. We will prove by induction. Let $P(n)$

be the claim "the transposition $(k\ k+n)$ can be expressed as a product of transpositions of the given form". Then $P(1)$ is trivial, as the transposition $(k\ k+1)$ is already in the proper form. Now suppose $P(i)$ holds. Since $(k\ k+i+1) = (k+i\ k+i+1)(k\ k+i)(k+i\ k+i+1)$, and we can substitute a product of tranpositions for the middle term to conclude $P(i+1)$.